Financial Intermediaries and the Yield Curve

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Abstract

I study the yield curve dynamics in a general equilibrium model with financial intermediaries facing financing constraints. The economy features a positive real term premium in equilibrium stemming from financing constraints that occasionally bind. A flat yield curve reduces intermediaries' incentives to engage in maturity transformation and therefore is associated with lower levels of credit. I show that this mechanism 1) is a plausible reason for why a flattening of the yield curve precedes recessions and 2) also rationalizes why the term structure of distributions of future real outcomes are negatively skewed when financial conditions are tight.

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One of the main lessons of the large body of research on the nexus between financial intermediation and the macroeconomy is that financial intermediaries face constraints that distort the allocation of goods and capital—hence affecting agents' marginal valuations. In this paper, I argue that the yield curve contains information about such distortions because long-term yields are, by definition, a forecast of the economy's marginal valuation (Alvarez and Jermann, 2005). In other words, intermediaries' constraints are a macro source of term premia which means long-term yields and intermediaries' balance sheet dynamics are closely related.

For this, I study a canonical general equilibrium intermediary asset pricing model to underscore the mechanism through which intermediaries' constraints cause a positive term premium. I show that the connection between intermediaries' constraints, marginal valuations, and long-term yields help us in rationalizing the salient properties of the U.S. real yield curve.[1] In particular, the model features highly non-linear real yields, with an average upward sloping real yield curve and highly volatile long-term yields, facts that have proven very difficult to match in representative agent models (Duffee, 2018). These results are purely driven by the fact that financial intermediaries face occasionally binding constraints. Indeed, if intermediaries were always unconstrained, then the yield curve would be flat and constant.

The mechanism is grounded in two main elements: Intermediaries operate with leverage in equilibrium and they face financing constraints. These two elements have been extensively studied in the macro-finance literature, but in this paper I focus the

[1] I focus the analysis on real yields because nonlinear dynamics in real yields are a key component of long term nominal yields (Duffee, 2018). Adding an exogenous inflation process is relatively straightforward—see Cox, Ingersoll and Ross (1985b).

analysis on the yield curve.[2] To obtain leverage in equilibrium, I follow Brunnermeier and Sannikov (2014), among others, and I assume intermediaries are more efficient in handling risky assets. That is, financial intermediaries issue short-term deposits to savers to fund positions in long-term risky assets and take advantage of their relatively better investment technology. However, intermediaries' positions in long-term assets can be constrained in certain states of the world due to agency problems, as in Gertler and Kiyotaki (2015). As a consequence, when intermediaries hit their constraints, they are forced to sell risky assets to less efficient savers and subsequently prices decline, the aggregate price of risk increases, financial intermediaries wealth deteriorates even further, which force intermediaries to reallocate their portfolios, and so on. This well-known feedback mechanism has important implications for the yield curve, as I detail next.

The presence of occasionally binding constraints implies the economy features a bimodal distribution: It spends the vast majority of time in a "normal regime," in which constraints are slack, risk premia are low, the real interest rate is low, and volatility of asset prices is moderate. When negative aggregate shocks occur, the economy can enter in a "crisis regime" in which financing constraints are binding. Here, intermediaries reallocate their portfolios and wealth is transferred to inefficient savers. This inefficiency pushes the consumption level persistently below the trend growth and, therefore, the real interest rate persistently increases as agents perceive the "crisis regime" as transitory—consumption level will recover its trend in the future. But this occurs precisely when the price of risk spikes, implying that real bond prices go down in states in which the marginal investor values those resources the most—a "crisis regime." Thus,

[2] Recent literature, reviewed below, has departed from the representative agent analysis of the yield curve, but without stressing the role of financing constraints—a salient characteristic of intermediaries.

real bonds carry an endogenously time-varying term premium and the yield curve is upward sloping on average due to the fact there is always a non-zero probability the economy can hit financing constraints.

Besides accounting for the salient properties of the yield curve (positive term premium and highly volatile long-term yields), I show the mechanism relating financial intermediary wealth and the yield curve rationalizes interesting macroeconomic phenomena. These exercises are useful in the sense that they show the mechanism in the model is consistent with evidence beyond the scope of the yield curve, therefore providing external validation of the key economic forces in the model.

First, there is ample reduced-form evidence indicating that a flattening of the yield curve (i.e., long-term yields equal or lower than short-term yields) is associated with lower future economic activity. I rationalize this evidence through the lens of the model by using the connection between the slope of the yield curve and the quantity of credit intermediated in the economy. In the model, a flattening of the yield curve is associated with lower levels of credit because there is not much incentive for intermediaries to engage in maturity transformation (i.e., borrow short-term to lend long-term) when the slope of the yield curve is low. I show empirically that the slope of the yield curve predicts fluctuations in credit, which is consistent with the model's prediction. I argue this mechanism is, at least partially, a reason for why a flattening of the yield curve precedes recessions: A flattening of the yield curve is associated with lower credit growth—and, potentially, lower economic activity.[3]

Second, recent literature has stressed the role of financial conditions in driving the distribution of real variables in the near future (Adrian, Boyarchenko and Giannone,

[3] I do not incorporate endogenous growth in the model.

2019; Giglio, Kelly and Pruitt, 2016). More precisely, when financial conditions deteriorate, the forecasted conditional distribution of GDP growth becomes more negatively skewed. Moreover, this distribution changes with the forecast horizon; there is a term structure of conditional distributions that changes over the forecast horizon. The conditional distributions is intimately related with the yield curve, because long-term yields are conditional expectations of future variables (a point estimate), while the forecasted distribution includes computing the entire distribution of future realizations. To rationalize the evidence, I compute the term structure of the conditional probability density function of consumption and intermediaries' wealth across the horizon. This is the model's theoretical counterpart of the estimated conditional distributions in, for example, (Adrian et al., 2019). I show the model captures the evidence relatively well: conditional on a state in which intermediaries are constrained (tight financial conditions), the term structure of conditional distributions of growth exhibit a negative skewness. The skewness in the term structure of conditional distributions becomes zero (or slightly positive) when conditioning to a state in which intermediaries are unconstrained (loose financial conditions).

Related literature. This paper relates to a strand of literature that has departed from the representative agent analysis of the yield curve. In this line, part of the literature has stressed the role of certain agents (arbitrageurs, intermediaries, etc.) in explaining the yield curve dynamics, typically in a partial equilibrium setup (Vayanos and Vila 2019; Greenwood and Vayanos 2014; Haddad and Sraer 2019). Relative to this literature, the contribution of this paper is to use a general equilibrium framework to study the role of financing constraints in driving the yield curve dynamics.[4]

The general equilibrium framework I build on (He and Krishnamurthy, 2013; Brunnermeier and Sannikov, 2014; Gertler and Kiyotaki, 2015, among many others) has been extensively studied in the macro-finance literature to answer a variety of questions, particularly after the Great Recession. For example, Gertler and Karadi (2011) study unconventional monetary policies; Van der Ghote (2021) studies the coordination of conventional and macroprudential policies; Maggiori (2017) extends the framework to study the risk sharing dynamics between countries that differ in their degree of financial development; Bigio and Schneider (2017) analyze the role of financing constraints and liquidity shocks in driving the equity premium. Relative to this literature, the contribution of this paper is to shift the focus away from stocks (or "capital") to the yield curve dynamics. In particular, I show that financing constraints play a crucial role in producing an endogenously time-varying real term premium. Additionally, I show the connection between the yield curve and financial intermediaries' wealth is important to understand why changes in the yield curve are associated with recessions, and also to understand why tight financing constraints imply a negatively skewed distribution of future economic outcomes.

1 Model

I present a general equilibrium model along the lines of He and Krishnamurthy (2013) and Brunnermeier and Sannikov (2014) and focus on the pricing implications for the

[4]Other papers have studied the yield curve in a general equilibrium setup with heterogeneous agents (e.g., Wang 1996; Schneider 2022; Drechsler, Savov and Schnabl 2018; Ehling, Gallmeyer, Heyerdahl-Larsen and Illeditsch 2018, among others) but without financing constraints.

yield curve. For simplicity, I abstract from inflation and highlight the real forces driving the yield curve dynamics. The model can be extended to include exogenous inflation process following the seminal work of Cox et al. (1985b), but the data suggest that persistent shocks to inflation explain a very small fraction of yields' variance Duffee (2018). Also, extending the model to include a production with AK technology and convex adjustment costs to investment would not affect the main conclusions.

Time is continuous and denoted by $t > 0$. Aggregate output, denoted by y_t, follows

$$\frac{dy_t}{y_t} = \mu dt + \sigma dW_t,$$

where parameters $\mu > 0, \sigma > 0$ are constants and W_t is a standard Brownian motion in a complete probability space (Ω, F, P) . The economy is populated by a continuum of savers (denoted by s) and a continuum of financiers (denoted by f). The main source of heterogeneity between f and s is that the former have a comparative advantage in operating risky assets over the latter—which implies f and s engage in borrowing and lending in equilibrium.

Agents can trade two classes of assets, namely long-term risky assets and short-term risk-free deposits. The long-term asset is in exogenous fixed supply and I denote its ex-dividend price by q_t. This asset pays a dividend y_t per unit of time if held by f, but ωy_t, $\omega < 1$, if held by s. That is, it is more costly for savers than financiers to operate this risky asset.[5] which is a simple way to obtain endogenous leverage in equilibrium.

The total return of investing in the long-term asset consists of the dividend yield

[5] This assumption is equivalent to assume that savers have to pay a cost to operate risky assets (Gertler and Kiyotaki, 2015). For tractability, I model savers' relative disadvantage as a wedge in the dividend yield (Brunnermeier and Sannikov, 2014), An alternative assumption would be to assume f are allowed to issue deposits with a positive spread over the short term rate Di Tella and Kurlat (2017).

plus the capital gains. For financiers, this is

$$dR_{f,t} = \frac{y_t}{q_t}dt + \frac{dq_t}{q_t},$$

while for savers the total return is

$$dR_{s,t} = \frac{\omega y_t}{q_t}dt + \frac{dq_t}{q_t}, \ \omega < 1.$$

Second, the short-term deposit account is in zero net supply and it yields a risk-free interest rate per unit of time, denoted by r_t. For simplicity, I solve the model with the generic long-term asset q_t and the short-term deposit account. Then introduce zero-coupon bonds of all maturities that are also in zero net supply. That is, zero-coupons are redundant in the construction of the equilibrium, but they are useful to characterize the economy's equilibrium yield curve.

Savers choose how much to consume and save in order to maximize their expected discounted utility. They can allocate their portfolios between risk-free deposits issued by financiers and risky assets. Their optimization problem can be written as

$$U_t = \max_{c_t, \theta_{s,t}} E_t \left[\int_t^\infty e^{-\rho(u-t)} \frac{c_u^{1-\gamma}}{1-\gamma} du \right],$$

subject to

$$\begin{aligned} dn_{s,t} &= \left[n_{s,t} r_t - c_t + q_t \theta_{s,t} \left(E_t \left[dR_{s,t} \right] - r_t \right) + T_t \right] dt + q_t \theta_{s,t} \sigma_{q,t} dW_t, \qquad (1) \\ n_{s,t} &\geq 0, \end{aligned}$$

where $n_{s,t}$ is the savers' net worth, $\theta_{s,t}$ is the holding of risky asset, c_t the consumption flow, and T_t the net transfers received from financiers' profits. Transfers are locally riskless because below I assume the dividend policy implemented by financiers is so.

Financiers are in charge of managing a financial intermediary firm. They operate this firm by issuing deposits to savers as well as using their own wealth, $n_{f,t}$, but they face financing constrains (described below). To avoid financiers growing out of their constraints,[6] I assume they pay dividends to savers following an exogenous Poisson process with intensity λ. After paying dividends, financiers receive a fraction $\overline{x}$ of total wealth to re-start the financial firm. Then, financiers' problem is to maximize the value of the firm (i.e., the expected discounted value of firms' wealth), that is

$$V_{f,t} = \max_{\theta_{f,t}} E_t \left[\int_t^{\infty} \frac{m_u}{m_t} \lambda e^{-\lambda(u-t)} n_{f,u} du \right] \tag{2}$$

subject to

$$\begin{aligned} dn_{f,t} &= \left[r_t n_{f,t} + q_t \theta_{f,t} \left(E_t \left[dR_{f,t} \right] - r_t \right) \right] dt + \theta_{f,t} q_t \sigma_{q,t} dW_t, & (3) \\ n_{f,t} &\geq 0, \\ V_{f,t} &\geq \kappa \theta_{f,t} q_t, & (4) \end{aligned}$$

where $m_t = e^{-\rho t} c_t^{-\gamma}$ is savers' marginal utility and $\theta_{f,t}$ is financiers' holdings of the risky asset. Financiers face a financing constraint, (4), that can be motivated with a standard agency problem. Specifically, I follow Gertler and Kiyotaki (2015) and assume the value of the financial intermediary firm has to be greater than a fraction of the assets the firm

[6] Recall financiers possess a technological advantage over savers, a force that pushes financiers to absorb all the wealth in the economy.

holds. This constraint operates as an endogenous leverage constraint.

I next define a competitive equilibrium.

Definition 1 (Competitive equilbrium) *A competitive equilibrium is a set of aggregate stochastic processes: prices* q_t, r_t*, policy functions for savers* $(\theta_{s,t}, c_t)$ *, policy functions for financiers'* $\theta_{f,t}$*, the value of the financiers' firm* $V_{f,t}$*, such that*

1. *Given prices,* $(\theta_{s,t}, c_t)$ *solves savers' problem*

2. *Given prices,* $(\theta_{f,t}, V_{f,t})$ *solves financiers' problem*

3. *Markets clear (long-term asset, consumption good, and short-term debt)*

$$\begin{aligned}
\theta_{s,t} + \theta_{f,t} &= 1, \\
c_t &= \omega \theta_{s,t} y_t + \theta_{f,t} y_t, \\
n_{f,t} + n_{s,t} &= q_t.
\end{aligned}$$

where the last equation (market clearing for short-term debt) is redundant due to Walras' Law, but is useful to explicit that wealth holdings add up to total wealth q_t*.*

Before turning to the solution of the model, it is useful to characterize agents' optimization problems with their first order conditions. For savers,

$$r_t = -E_t \left[\frac{dm_t}{m_t} \right],$$

and

$$E_t [dR_{s,t}] - r_t dt \leq -E_t \left[\frac{dm_t}{m_t} dR_{s,t} \right]$$

with equality if households are holding long-term assets (i.e., $\theta_{s,t} > 0$). The optimality conditions for financiers requiere a few more steps, and it is useful to first write financiers' problem in a recursive way. First, notice that due to the linearity of financiers' objective function and constraints, the value function can be written as[7]

$$V_{f,t} = \psi_t n_{f,t}, \tag{5}$$

where $\psi_t \geq 1$ is an endogenous Ito process whose drift $\mu_{\psi,t}$ and diffusion $\sigma_{\psi,t}$ are solved in equilibrium. Then the financing constraint can be written as

$$\begin{aligned} \psi_t n_{f,t} &\geq \kappa \theta_{f,t} q_t, \\ \psi_t &\geq \kappa \frac{\theta_{f,t} q_t}{n_{f,t}} \equiv \alpha_{f,t}, \end{aligned}$$

where $\alpha_{f,t}$ is the endogenous financiers' portfolio share in the risky asset. The financiers' problem can be written in a recursive way as (i.e., the Hamilton-Jacobi-Bellman, HJB)

$$0 = \max_{\theta_{f,t}} \lambda \left(n_{f,t} - V_{f,t} \right) m_t dt + E_t \left[d \left(m_t V_{f,t} \right) \right] + \chi_t \left(V_{f,t} - \kappa \theta_{f,t} q_t \right) dt. \tag{6}$$

where χ_t is the Lagrange multiplier associated with the financing constraint. Using (5) in (6), the first order conditions for financiers can be written as

$$E_t \left[dR_{f,t} \right] - r_t \geq -E_t \left[\left(\frac{dm_t}{m_t} + \frac{d\psi_t}{\psi_t} \right) dR_{f,t} \right]$$

with equality if $\chi_t = 0$. Put differently, financiers are the marginal investors in long-term

[7] See Gertler and Kiyotaki (2015).

risky assets if their constraints are not binding. If financing constraints are binding, then their holdings in risky assets are pinned down by such constraints (i.e., $\psi_t = \kappa\alpha_{f,t}$), and savers are the marginal investors in risky assets.

Yield curve. Finally, I characterize the yield curve in the economy, which consists of the endogenous price vector $\left\{P_t^{(\tau)}\right\}_{\tau\geq 0}$. Yields can then be obtained simply as $y_t^{(\tau)} = -\log P_t^{(\tau)}/\tau$. The zero-coupon bonds are risky assets in the sense that they convey a premium in equilibrium (i.e., there are endogenous fluctuations in the interest rate and in the price of risk). As with the long-term risky asset, savers are the marginal investor in the zero coupon bond when financiers are constrained. When financiers are unconstrained, their risk bearing capacity is high enough and they are the marginal investor. That is, by no-arbitrage, the expected excess return of zero-coupon bonds is

$$E_t\left[\frac{dP_t^{(\tau)}}{P_t^{(\tau)}}\right] - r_t dt = \begin{cases} -cov_t\left(\frac{dm_t}{m_t}, \frac{dP_t^{(\tau)}}{P_t^{(\tau)}}\right) & \text{if } \chi_t > 0, \\ -cov_t\left(\left(\frac{dm_t}{m_t} + \frac{d\psi_t}{\psi_t}\right), \frac{dP_t^{(\tau)}}{P_t^{(\tau)}}\right) & \text{if } \chi_t = 0, \end{cases}$$

which is the same pricing equation as for the generic long-term asset used in the construction of the equilibrium.

Implementing a bond portfolio. Rather than holding the generic long-term asset q, financiers may hold a relatively more standard portfolio consisting of long-term zero coupon bonds on their asset side, funded by short-term deposits on their liability side. I now compute the implied maturity of those long-term zero-coupon bonds to match the risk exposures chosen in equilibrium in the model. This exercise is useful to understand the extent to which the implied maturity mismatch in zero-coupon bonds is in line with with the observed traditional maturity mismatch of financial intermediaries.

For this, I follow Di Tella and Kurlat (2017) and assume financiers issue a constant fraction of their net worth in the form of short-term deposits. They use those resources to invest in a zero-coupon bond of maturity τ, with price $P_t^{(\tau)}$. I denote the fraction of deposits as $\phi n_{f,t}$, and thus $(1+\phi)n_{f,t}$ is the exposure to the zero-coupon bond. That is, $n_{f,t} = x_{f,t}^{(\tau)} - b_{f,t}$ where $x_{f,t}^{(\tau)}$ is the value of the financier's holding in the zero-coupon bond of maturity τ. Then, $b_{f,t} = \phi n_{f,t}$, we have that the exposure to the zero-coupon bond is $(1+\phi)n_{f,t} = x_{f,t}^{(\tau)}$. From this expression, I can then solve for the maturity τ that matches the bank exposure to the long-term asset with price q. This means

$$\frac{\theta_{f,t} q_t}{n_{f,t}} \sigma_{q,t} = (1+\phi)\, \sigma_t^{(\tau)}, \tag{7}$$

where $\sigma_t^{(\tau)}$ is the volatility of a zero-coupon bond with maturity τ. Then, the τ that solves (7) is the maturity that implements the risk exposure chosen by financiers (under the business model that assumes a constant fraction of net worth in the form of deposits). This chosen maturity may change over the state-space, as I discuss in the results.[8]

2 Model Solution

I use the homogeneity property of objective functions and constraints to solve the equilibrium in a recursive fashion, using a single endogenous state variable,

$$x_t = \frac{n_{f,t}}{q_t} \in [0,1]\,. \tag{8}$$

[8] Alternatively, although it is redundant, I could fix a maturity τ and solve for the exposure ϕ_t that mimics the financiers exposure to long-term assets.

The endogenous state variable x_t follows an Ito process with drift $x\mu_{x,t}$ and diffusion $x\sigma_{x,t}$. The objective is to characterize the equilibrium with the optimality conditions for savers and financiers as a function of x. That is, I solve for financiers' marginal value, $\psi(x_t)$, rescaled risky asset $p(x_t) = q_t/y_t$, short-term risk free rate $r(x_t)$, zero-coupon bonds $\{P(x_t,\tau)\}_{\tau\geq 0}$, and financiers' portfolio share $\alpha_f(x_t)$. Then, the equilibrium can be characterized by a system of non-linear ordinary differential equations on the state variable x_t. In particular, I use the financiers' Hamilton-Jacobi-Bellman equation together with the asset pricing condition for savers when financiers are constrained and the asset pricing condition for financiers when they are unconstrained. Thus, a critical element for the solution is to find the value $x_t^* \in [0,1]$ at which financiers become constrained. The next proposition shows the system of equations.

Proposition 2 *The Markov equilibrium is characterized by the following system of ordinary differential equations. In the unconstrained region (i.e., $x > x^*$) the system is*

$$
\begin{aligned}
0 &= \frac{1}{p(x)} + \mu_p(x) + \mu + \sigma_p(x)\sigma - r(x) + \left(\sigma_\psi(x) - \gamma\sigma_c(x)\right)\left(\sigma_p(x)+\sigma\right), \\
0 &= \frac{\lambda(1-\psi(x))}{\psi(x)} + \mu_\psi(x) - \gamma\sigma_c(x)\sigma_\psi(x), \\
0 &= -\frac{P'_\tau(x,\tau)}{P(x,\tau)} + \mu_P(x,\tau) + \frac{1}{2}\sigma_P(x,\tau)^2 - r(x) + \left(\sigma_\psi(x) - \gamma\sigma_c(x)\right)\sigma_P(x,\tau),\ P(x,0) = 1\ \forall x.
\end{aligned}
$$

In the constrained region (i.e., $x \leq x^*$) the system is

$$
\begin{aligned}
0 &= \frac{\omega}{p(x)} + \mu_p(x) + \mu + \sigma_p(x)\sigma - r(x) - \gamma\sigma_c(x)\left(\sigma_p(x)+\sigma\right) \\
0 &= \frac{\lambda(1-\psi(x))}{\psi(x)} + \frac{\psi(x)}{\kappa}\left(\frac{(1-\omega)}{p_t} + \sigma_\psi(x)\left(\sigma_p(x)+\sigma\right)\right) + \mu_\psi(x) - \sigma_\psi(x)\gamma\sigma_c(x) \\
0 &= -\frac{P'_\tau(x,\tau)}{P(x,\tau)} + \mu_P(x,\tau) + \frac{1}{2}\sigma_P(x,\tau)^2 - r(x) - \gamma\sigma_c(x)\sigma_P(x,\tau),\ P(x,0) = 1\ \forall x
\end{aligned}
$$

where for a given function $z(x)$, the diffusion $\sigma_z(x)$ and drift $\mu_z(x)$ are in geometric form,

$$\begin{aligned}
\frac{dz_t}{z_t} &= \mu_{z,t}(x_t)\,dt + \sigma_{z,t}(x_t)\,dW_t \\
&= \left[\frac{z'_x}{z} x\mu_x(x) + \frac{1}{2}\frac{z''_{xx}}{z}\left(x\sigma_x(x)\right)^2\right] dt + \frac{z'_x}{z} x\sigma_x(x)\,dW_t.
\end{aligned}$$

The point x^* is such that $\forall x > x^*$, $\frac{\psi(x)}{\kappa} > \alpha(x)$ (constraint is slack) and $\forall x < x^*$ $\frac{\psi(x)}{\kappa} = \alpha_f(x)$ (constraint is binding). The real interest rate, $r_t = r(x_t)$, is $r_t = -E_t\left[\frac{dm}{m}\right]$, where $m_t = e^{-\rho t} c_t$.

Proof. See appendix.

3 Results

Calibration. I calibrate the model at an annual frequency and solve it a global solution technique based on projection methods (see appendix). Table 1 shows the parameters. As I highlight below, risk aversion, γ, plays a critical role in the model solution. I set γ=5, which is in the range of the standard values used in the asset pricing literature. Lower values of risk aversion alleviate the non-linearity produced by the occasionally binding constraints. I set σ=0.034, a number that is in line with the volatility of productivity in the US and close to the value used in He and Krishnamurthy (2019).[9] I set parameters ρ to pin down a level of the short rate that is consistent with the evidence. Lastly, I set μ to target an aggregate real consumption growth of 1.5%.

The remaining parameters are associated with financiers' technology and constraint.

[9] Previous papers have used a wide range of values for σ. For example, He and Krishnamurthy (2013), in a similar setup, uses σ=0.09; Brunnermeier and Sannikov (2014), also in a similar setup but with endogenous production, uses σ=0.1.

I calibrate κ=0.4 to target an average leverage of 3 (He and Krishnamurthy, 2019). I set λ=0.08, which gives an expected payout rate of the intermediary as in Gertler and Kiyotaki (2015). I set ω=0.85 which implies asset prices can drop no more than 50% across the state space (i.e., when savers hold the entire wealth in the economy, the price-dividend ratio is 50% lower than if intermediaries held the entire wealth in the economy). This assumption yields a conservative lower bound of changes in the price-dividend ratio. Lastly, I set $\overline{x} = 0.2$ to stabilize the wealth of intermediaries below 0.5.

Solution and mechanism. Figure 1 show the solution of the key endogenous variables. Both figures display the endogenous variables in the Markov equilibrium (i.e., endogenous variables as a function of the state variable x). The red dashed line in all panels represents the point at which the financing constraint binds.

The invariant distribution, displayed in lower left panel of Figure 1, shows the economy has two modes. It spends the majority of the time in a normal regime where constraints are slack (i.e., to the right of the red dashed line), and some time in a crisis regime, where constraints are binding. Normal times are characterized by low volatility, low rates, and moderate leverage. As is common in these types of models, leverage is counter-cyclical: The lower the intermediaries wealth is (i.e., lower x), the higher the leverage.

If the economy is in the normal regime and a sufficiently negative aggregate shock occurs, financial intermediaries reallocate their portfolios, the price of risky assets declines, and the price of risk increases. Financing constraints may bind (depending on the magnitude of the shock) and trigger the well-known financial accelerator mechanism studied in previous literature (e.g., Bernanke, Gertler and Gilchrist (1999)), in which lower valuations deteriorate intermediaries' wealth even further.

A central element in the yield curve dynamics is the behavior of the short-term interest rate, r. Notice that when the economy enters in a crisis regime, the price of risk spikes and the real interest rate increases. This is because wealth is transferred to savers, who are inefficient in handling risky assets, which means the level of aggregate dividends (and consumption) declines. Because the inefficiencies caused by the mis-allocation of risky assets are temporary, savers expect consumption level to increase in the future, which increases the real interest rate. Put differently, the dynamics for consumption level are similar to a random-walk with drift, where deviations from the trend are persistent. When consumption is below the trend, it is expected to mean-revert in the future. In the model, the trend is endogenously driven by financial intermediaries wealth dynamics.

The yield curve. Intuitively, investors require a premium to hold an asset whose value persistently declines in states in which the price of risk is high. This is precisely what drives the real term premium in the economy: real bond prices decline (i.e. real rate persistently increases) in states in which the price of risk is high. Figure 2 shows the average yield curve in the economy. Simply put, in the stochastic steady state—where expected short rates are constant—long-terms yields are driven by the term premium, causing the yield curve to be upward sloping on average. The left panel of Figure 2 illustrates the dynamics of yields at different horizons across the state space. The mechanism through which financial intermediaries reduce their positions in risky assets by selling those to less efficient savers, is noticeable only in short maturity rates—long-term yields are less sensitive to the missallocation of wealth in the economy. Put differently, current fluctuations in financiers' wealth has a lower incidence in driving longer maturity bonds, a feature that can be appreciated in the left panel of Figure 2. The panel shows the yield of bonds at 1, 10, and 30 year maturities and also displays the yield of a very long-term bond. As the horizon of the bond increases, the yields become less sensitive to current financial conditions: x_t has a smaller impact on yields' dynamics. This result, driven by the persistence and stationarity of x, shows that even very long term rates can display substantial volatility.

Figure 3 shows the yield curve for different levels of x. The circles in the Figure represents the average real yields reported in Backus, Boyarchenko and Chernov (2018). The yield curve is flat when x is high mainly because term premiums and real rates are low. Intuitively, a high x is a state in which intermediaries are relatively well capitalized, all term premium opportunities have been exhausted, and financing constraints are slack. When x is low, however, the economy is in crisis times, constraints are binding and real yields are high. In this state, the short term rate is expected to mean revert, and this force pushes down long-term rates (even though term premia is high, the expectations of the short-rate dominates). Thus, the gray line shows a downward sloping yield curve.

Implementation. I now explore how financial intermediaries could replicate their risk exposures using a portfolio consisting of a deposits and a zero-coupon bond of a given maturity. This exercise is useful to compute banks' average maturity mistmatch in the model and compare that with the data. As shown in English, Van den Heuvel and Zakrajšek (2018), banks display an average maturity mismatch of approximately 5 years, which, as shown below, it is close to the prediction of the model (6.4 years).

Figure 4 shows the implementation of a portfolio of zero-coupon bonds, following equation (7). The idea is to find the maturity of a zero-coupon bond that financiers can use to replicate the same risk exposure as the one they choose using the long-term asset q. Following Di Tella and Kurlat (2017), I assume financiers' exposure to the zero-coupon bond is constant, so they issue a constant fraction ϕ of their net worth in the form of short-term deposits. Then $1+\phi$ is the exposure to the zero-coupon bond. I use ϕ=8.77 as in Di Tella and Kurlat (2017).[10] The top panel shows the volatility of zero-coupon bonds across the state space, for three different maturities. As expected, the larger the horizon of the bond, the larger the volatility. Volatility of bonds (and more generally of asset prices) display a non-linear shape. After constraints bind, financiers' balance sheets contract and, therefore, their aggregate pricing effect is reduced. The bottom panel shows the corresponding maturity of the zero-coupon bond that replicates financiers' risk exposure in the long-term asset q. Without loss of generality, I assume financiers have available bonds of any maturity up to 30 years. On average, the chosen maturity is 6.4 years, but this value changes across the state space (and so the volatility of long-term bonds). This prediction in the model indicates that intermediaries' maturity exposure is time-varying and negatively correlated with the volatility of long-term bonds.

The role of risk aversion. Risk aversion plays a crucial role in the equilibrium dynamics. Figure 5 shows that when γ=2, the yield curve is almost flat, the price of risk is low and does not move much, and real interest rates are less volatile. Financing constraints, however, bind at a similar endogenous level to the baseline calibration of γ=5. That is, the red dashed line, which is the point at which financing constraints bind in the baseline calibration is close to the black dashed line. Notice that the lower level of risk aversion implies that the invariant distribution is not bimodal as in the baseline, but instead shows a single mode. The economy spends the vast majority of the time in a constrained region.

[10] Di Tella and Kurlat (2017) set ϕ=8.77 to match banks' deposits to net-worth ratio in 1990-2014.

When savers are less risk-averse, the real rate has to fluctuate much less to clear the deposit markets. Also, the price of risk demanded by intermediaries is lower, because they discount prices with savers' marginal utility. This feature implies intermediaries' balance sheets are less volatile with a lower level of γ (i.e., their liability side fluctuates much less), which in turn affects the volatility of asset prices and the yield curve dynamics—intermediaries' wealth and asset prices are endogenously determined.

Discussion: The real interest rate during financial recessions. The model has implications for the cyclical properties of real rates when financing constraints tighten, which can be associated to financial recessions. I next discuss the evidence for real real rates in such episodes.

As discussed above, the model implies that when financing constraints bind, the real interest rate persistently increases on impact —which means real bond prices decrease. Because this is a state of the economy in which the price of risk increases, real bonds command a positive premium to incentivize the marginal agent to hold them. Put differently, real bonds prices go down precisely in states in which the marginal agent value resources the most. This implies there is a positive real term premium and an average upward sloping real term structure.

In the evidence I discuss next, I illustrate that real interest rate increases during financial recessions because these are events in which inflation expectations move down much faster than nominal interest rates. I provide evidence about TIPS and ex-post real rates in the U.S. during the Great Recession, Gilts and ex-post real rates in the U.K. during the Great Recession. Finally, I study the evidence of the ex-post real interest rate in 14 advanced economies during the last 150 years using the dataset in Jordà, Schularick and Taylor (2016).

TIPS experienced a pronounced increase during the Great Recession. For example, 1-year TIPS went from about 0% to almost 7%. Longer maturity TIPS which are considered to be more liquid than the 1-year rate, also increased sharply (from levels close to zero to almost 3%). Anecdotal evidence suggests that a large fraction of this sharp increase could be associated with liquidity issues rather than deflation fears. To avoid using market data for TIPS, the upper-right panel shows the ex-post real rate. This is the short-term nominal interest rate minus realized inflation. On impact, the ex-post real rate increases because the decline in nominal rates was smaller than the decline in the realized inflation during 2008. During 2009 inflation turned out to be less negative than expected (i.e., deflation fears were not confirmed[11]) and that is what explains the ex-post real interest rate decline in the second part of the recession. The bottom two panels display the same evidence (real bonds and ex-post real rates) for the U.K. The dynamics are very similar to those experienced in the U.S., particularly for real bonds.

In order to have more comprehensive evidence about the dynamics of real rates during financial recession, I next use the data provided by Jordà et al. (2016) for 14 advanced economies during the last 150 years.[12] Panel A of Table 3 displays the evidence for all the 14 advanced economies in the Full Sample (1850-2016) and the post-1900 sample. Panel B focuses only in the U.S.[13]

The peak of the financial recession is labeled as t, therefore $t+1$ represents the year after the peak when the actual financial recession has already hit the economy. As it can be seen from both panels, in all samples, the ex-post real rate actually increases

[11] In fact, December 2009 showed a 2.8% year-over-year inflation reading.

[12] Ideally one would like to compute the ex-ante real interest rate but data for inflation expectations is rather limited.

[13] The financial recessions for the U.S. are 1873, 1882, 1892, 1906, 1929, 2009; see Jordà et al. (2016) for more details

on impact. This is, as shown, because the nominal interest rate decreases but realized inflation decreases relatively more. Through the lens of the Fisher identity (nominal rate equal real rate plus expected inflation) and assuming agents have perfect foresight (which implies realized and expected inflation are equal), this implies the real interest rate increases.

Credit cycle and the yield curve. A relatively well-known empirical regularity is that a flattening of the yield curve is associated with lower future economic growth. The model presented in the previous section predicts there is a link between yields and the real economy is the amount of credit intermediated. Although the model does not include an explicit production sector, a basic extension can be added to link credit and production. A plausible explanation for why the yield curve precedes recessions is that the yield curve fluctuates, at least partially, with aggregate credit. Thus, a lower term spread reduces financers' incentives to engage in maturity transformation, which creates a reduction of the aggregate amount of credit.

Figure 6 shows the model's prediction for the relationship between credit and the slope of the yield curve. The positive correlation between the slope and credit indicates that states in which the slope is large correspond to expansions in credit. Intuitively, this correlation is driven by the fact that a larger term spread makes it more profitable to engage in maturity transformation and therefore expand credit.

To test the positive relationship between the slope of the yield curve and credit, I run the following regression:

$$\Delta credit_t = \alpha + \beta \Delta slope_{t-1} + \boldsymbol{\gamma' \kappa_{t-1}} + \varepsilon_t.$$

Credit is the log-difference of real total loans to non-financial private sector between t

and $t-1$, the slope is the difference between long and short interest rates in $t-1$, and $\boldsymbol{\kappa}_{t-1}$ are controls.[14] The source of the data is Jordà et al. (2016) and I run the regressions in three subsamples.

Table 2 shows the results. The slope of the yield curve is associated with the evolution of total credit in the economy. This positive association between credit and the slope of the yield curve is robust across subsamples and also after controlling for GDP growth or changes in the slope. This positive association indicates that when the slope of the yield curve is small, the incentives to engage in maturity transformation are reduced, resulting in a contraction in credit. This force is a plausible explanation for why the slope of the yield curve precedes recessions: It anticipates a contraction in credit.

Term structure of conditional distributions. Recent literature has been stressing the role of financing conditions in forecasting the distribution of future real variables (Adrian et al., 2019). The term structure of distributions is related to the yield curve because the former is a forecast of the conditional distribution of a random variable —the marginal utility in the economy—at a certain point in the future, while the latter is the expected value of a given payoff at certain point in the future.[15] I show the key economic forces driving the yield curve, elaborated above, is also consistent with the evidence about the term structure of conditional distributions of future outcomes.

To compute the term-structure of distributions, consider a process z_t in the model, that follows an Ito process

$$dz_t = \mu_{z,t}dt + \sigma_{z,t}dW_t,$$

[14] I take the difference because the level of credit is nonstationary in the data. Alternatively, one could use credit/GDP and results would be similar.

[15] In technical terms, these two objects are the forward Kolmogorov equation and the backward Kolmodorov equation.

where $\mu_{z,t} = \mu_z(x_t)$ and $\sigma_{z,t} = \sigma_z(x_t)$ are the drift and diffusion. Next, I define the function $f(x_s|x_t = x^*, s)$ as the conditional distribution of x at each point in time $s > t$, starting from a point x^*. The evolution of the density over time can be described by the following partial differential equation

$$\frac{\partial f(z(x)|x^*,t)}{\partial t} = -\frac{\partial}{\partial x}\left[f(z(x)|x^*,t)\,\mu(x)\right] + \frac{1}{2}\frac{\partial^2}{\partial x^2}\left[f(z(x)|x^*)\,\sigma(x)^2\right],$$

which is also known as the forward Kolmogorov equation (or Fokker-Planck equation).

Figure 8 shows the forecasted conditional distributions for log consumption growth (top two panels) and the state variable x (bottom two panels) at different horizons. The blue line represents the forecasted density conditional on current financial conditions being loose. More precisely, the forecast is conditional on $x_t = x^*$ where x^* is 10% above the point at which financing constraints bind. The distribution in red represents the forecasted density conditional on current financial conditions being tight. I assume x_t is 10% below the point where financing constraints bind. The red dashed line in all four panels represents the point in the state space at which financing constraints bind.

In line with the evidence reported in Adrian et al. (2019), the top two panels indicate that, conditional on the economy facing tight financial conditions, the distribution of future growth is negatively skewed. Also, the conditional distribution fluctuates across the horizon, and the relatively smaller skewness of the conditionally constrained distribution persists. The main source of the asymmetry is that economic outcomes are quite different in the constrained and unconstrained regions. For example, in the constrained region the economy is more leveraged (thus more sensitive to shocks), the real rate is much more volatile, and the price of risk moves faster. These conditions may persist because it takes time to financiers' wealth to be rebuilt. Simply put, x is a persistent process.

The bottom two panels display the forecasted conditional distributions for the state variable x. The intuition is similar to that of consumption growth. Tight financial conditions are persistent and can trigger quite volatile and unstable outcomes. Simply put, the model rationalizes the data with two main elements: Tighter financial conditions are persistent outcomes and they lead to quite different economic outcomes than those implied by the economy functioning in an unconstrained region. As with the yield curve, the key elements are the bimodal nature of the economy and the persistent dynamics of intermediaries' wealth.

4 Conclusion

Financial intermediaries hold long-term assets, which means fluctuations in the long-term yields affect the extent to which intermediaries are financially constrained. These constraints affect marginal valuations in the economy, not only of the intermediaries, but in general equilibrium they could affect other agents' marginal valuations as well. Hence, because long-term yields are forecasts of marginal valuations, financing constraints and long-term yields are directly related with each other.

Indeed, I show that financing constraints generate an endogenously time-varying real term premium that is consistent with the data. The mechanism I propose can rationalize relevant yield curve facts, such as an upward sloping real yield curve and highly volatile long-term yields, which are indeed hard to capture in standard macro models Duffee (2018). These results are purely driven by the fact that intermediaries financing constraints may occasionally bind, linking the yield curve with intermediaries' financial health.

The novel economic mechanism I propose, connecting intermediaries' wealth and the yield curve, can rationalize interesting macroeconomic phenomena, suggesting that there are several potential avenues for further research. In particular, I show that a flattening of the yield curve precedes recessions because the slope of the yield curve is connected to intermediaries' willingness to engage in maturity transformation (and hence credit supply). Additionally, I show that the same mechanism explaining the term structure of interest rates is also able to rationalize the negative skewness in the term structure of distributions of consumption growth when financing constraints are binding.

TABLE 1. Calibration

PARAMETERS

	Value	Description
γ	5	risk aversion
ρ	0.0001	time preference
σ	0.034	volatility y_t
μ	0.007	drift y_t
λ	0.08	dividend payout
ω	0.85	managment cost
κ	0.4	fraction divertible assets

NOTES: This table shows the calibration of the model at an annual frequency.

TABLE 2. Slope of the yield curve and credit

	Dependent variable: $\Delta credit_t$					
	Full sample (1870-2016)			Postwar (1946-2016)		
Regressors	(1)	(2)	(3)	(4)	(5)	(6)
$\Delta slope_{t-1}$	0.922**	0.909**	0.745**	1.182**	1.172**	0.831*
	(0.386)	(0.421)	(0.366)	(0.575)	(0.613)	(0.450)
$\Delta credit_{t-1}$	0.598***	0.600***	0.609***	0.517***	0.523***	0.510***
	(0.091)	(0.081)	(0.083)	(0.176)	(0.155)	(0.146)
Δgdp_{t-1}	–	-0.014	-0.027		-0.034	-0.070
		(0.897)	(0.110)		(0.228)	(0.200)
$slope_{t-1}$	–	–	0.323*			0.607**
			(0.187)			(0.278)
R^2	0.344	0.344	0.349	0.311	0.316	0.345
Obs.	135	135	135	70	70	70

NOTES: The source of the data is Jordà et al. (2016). The dependent variable is $\Delta credit_t$, the log difference of real credit (Total loans to non-financial private sector over CPI) from $t-1$ to t. All regressions from (1) to (6) include a constant (not reported). Heteroskedasticity- and autocorrelation consistent asymptotic standard errors reported in parentheses are computed according to Newey and West (1987) with the automatic lag selection method of Newey and West (1994): *p<0.10; **p<0.05; and ***p<0.01.

TABLE 3. Ex-post real interest rate during financial recessions

A) All Countries

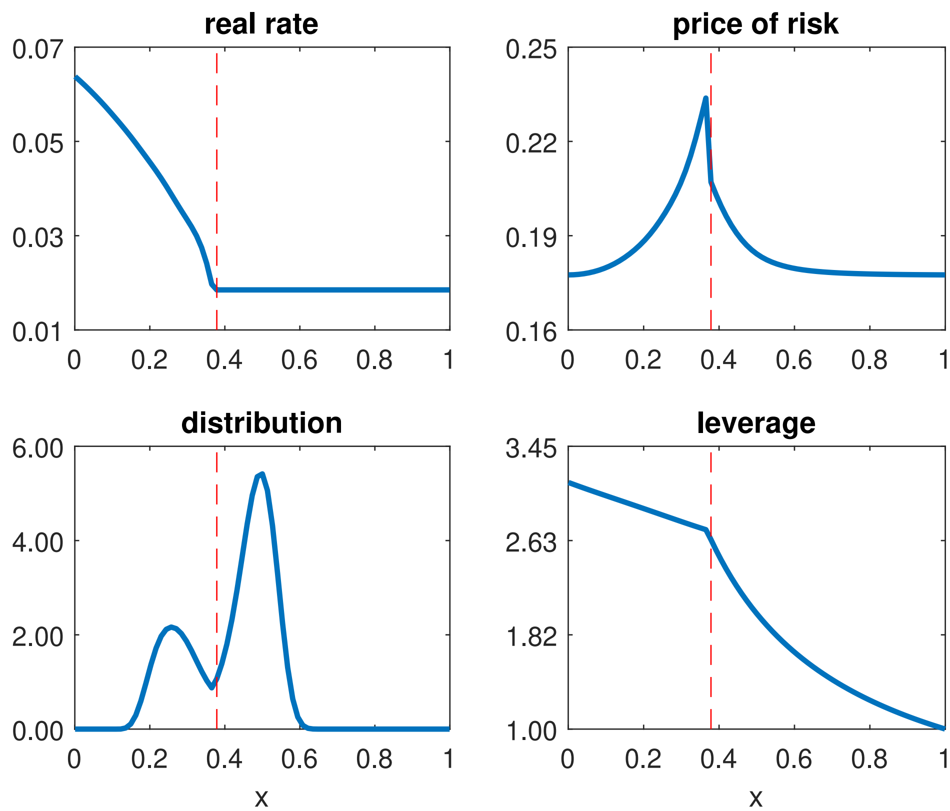
	r			i			π		
Full sample (N=63)	mean	median	st.dev	mean	median	st.dev	mean	median	st.dev
$t-1$	0.035	0.035	0.053	0.053	0.047	0.031	0.018	0.016	0.053
t	0.057	0.046	0.073	0.058	0.046	0.038	0.004	0.008	0.067
$t+1$	0.059	0.054	0.058	0.051	0.043	0.032	-0.007	-0.003	0.063
Post 1900 (N=42)									
$t-1$	0.017	0 .021	0.071	0.055	0.050	0.034	0.037	0.023	0.050
t	0.056	0.047	0.060	0.063	0.050	0.044	0.007	0.011	0.068
$t+1$	0.060	0.054	0.077	0.056	0.049	0.038	-0.004	-0.001	0.073

B) U.S. only

	r			i			π		
Full sample (N=6)	mean	median	st.dev	mean	median	st.dev	mean	median	st.dev
$t-1$	0.050	0.051	0.022	0.055	0.054	0.016	0.004	0.000	0.011
t	0.076	0.056	0.069	0.069	0.058	0.039	-0.001	-0.012	-0.007
$t+1$	0.078	0.078	0.034	0.039	0.036	0.017	-0.039	-0.036	-0.039
Post 1900 (N=3)									
$t-1$	0.042	0.044	0.019	0.052	0.049	0.008	0.009	0.000	0.016
t	0.047	0.029	0.047	0.064	0.065	0.013	0.016	0.036	0.036
$t+1$	0.083	0.106	0.053	0.039	0.029	0.026	-0.043	-0.036	0.044

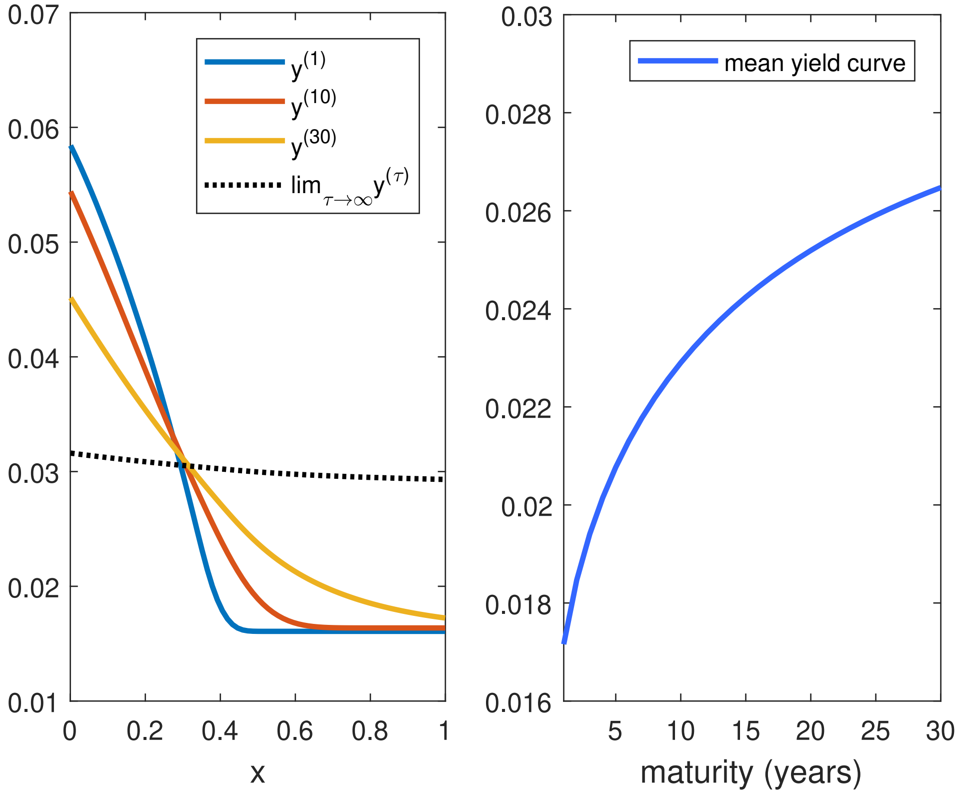
NOTES: This table shows the ex-post real rate (r), the nominal rate (i), and inflation (π) for 14 advanced economies (panel A), the U.S. (panel B) during financial recessions as defined by Jordà et al. (2016). The label t corresponds to the peak of the cycle associated with the financial.

FIGURE 1. Model solution



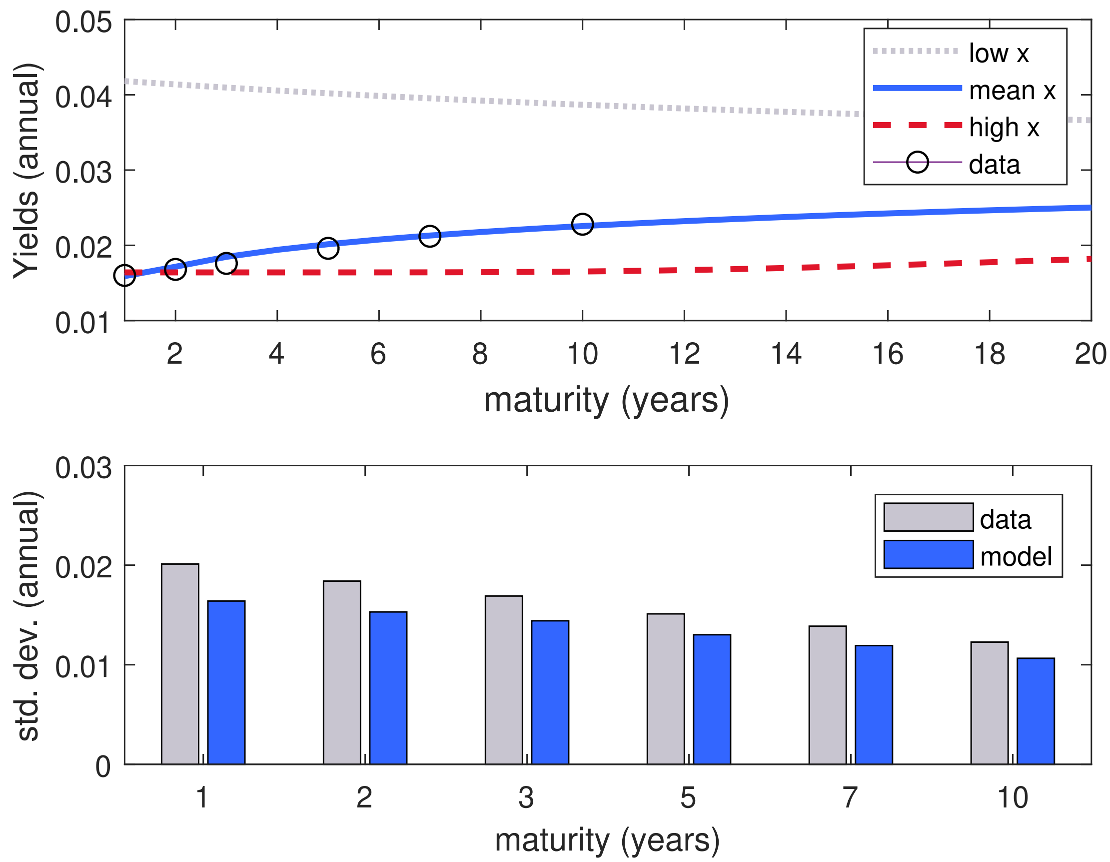
NOTES: This figure shows the model solution, with the calibrated parameters from Table 1 .

FIGURE 2. Model solution: Yields



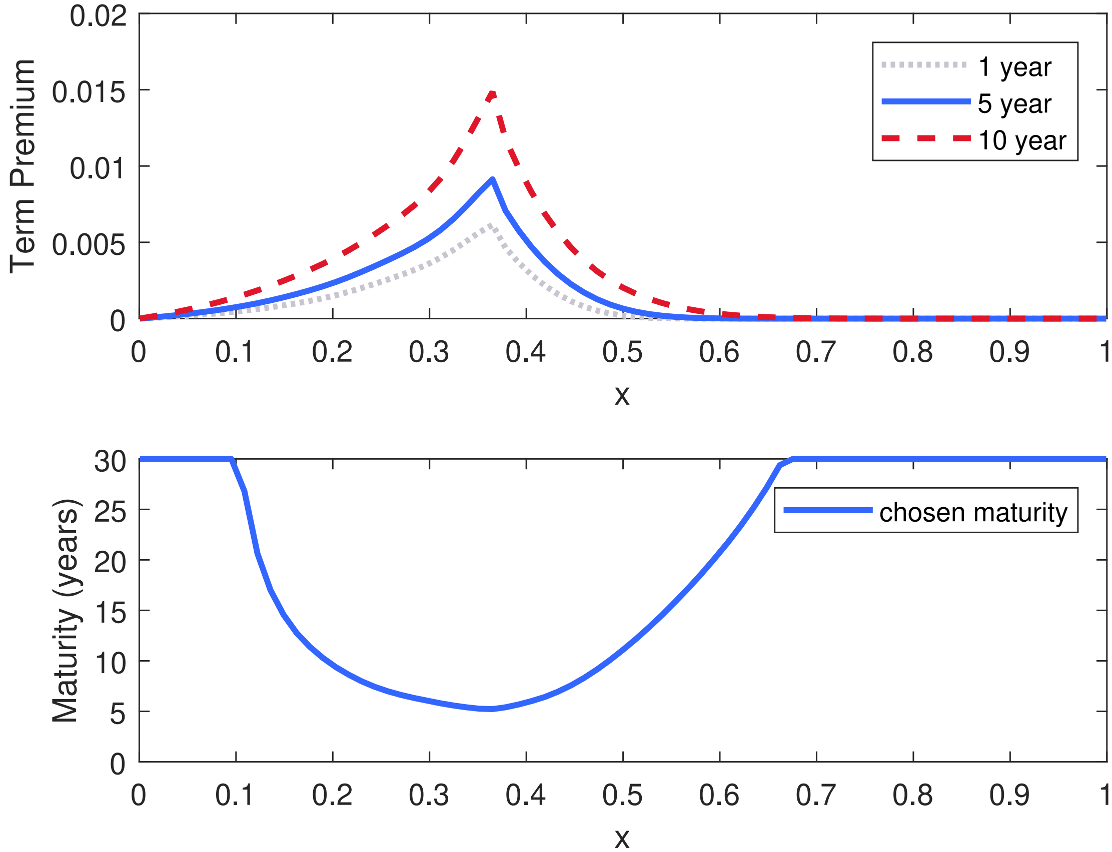
NOTES: This figure shows the model solution, with the calibrated parameters from Table 1 .

FIGURE 3. Yield Curve: First and Second Moments



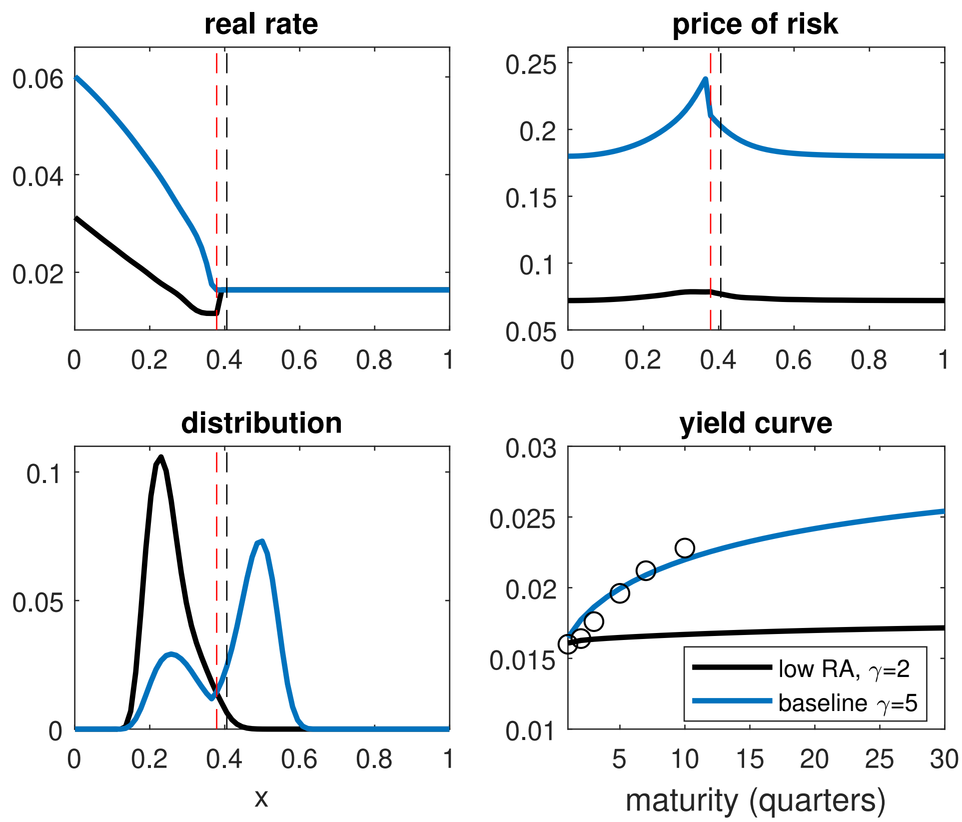
NOTES: This figure shows the yield curve in the model, with the calibrated parameters from Table 1. The top panel displays the yield curve for three different levels of x (mean, and +- 2 standard deviations). The bottom panel show the standard deviation of yields. The data for real yields from Backus et al. (2018), updated until 2018Q4.

FIGURE 4. Implementation of bond portfolio



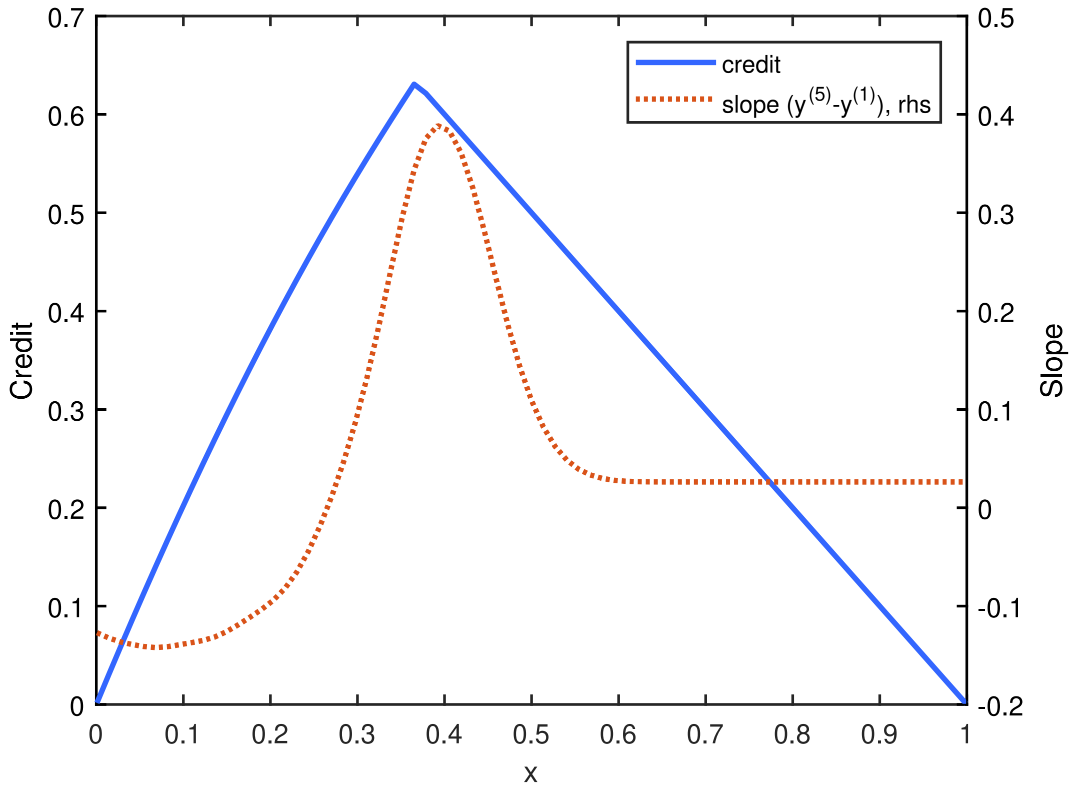
NOTES: The top panel shows the term premium of zero-coupon bonds of different maturity. The bottom panel shows the maturity of the zero-coupon bond portfolio that matches the risk exposure of financiers to long-term assets.

FIGURE 5. The role of risk aversion



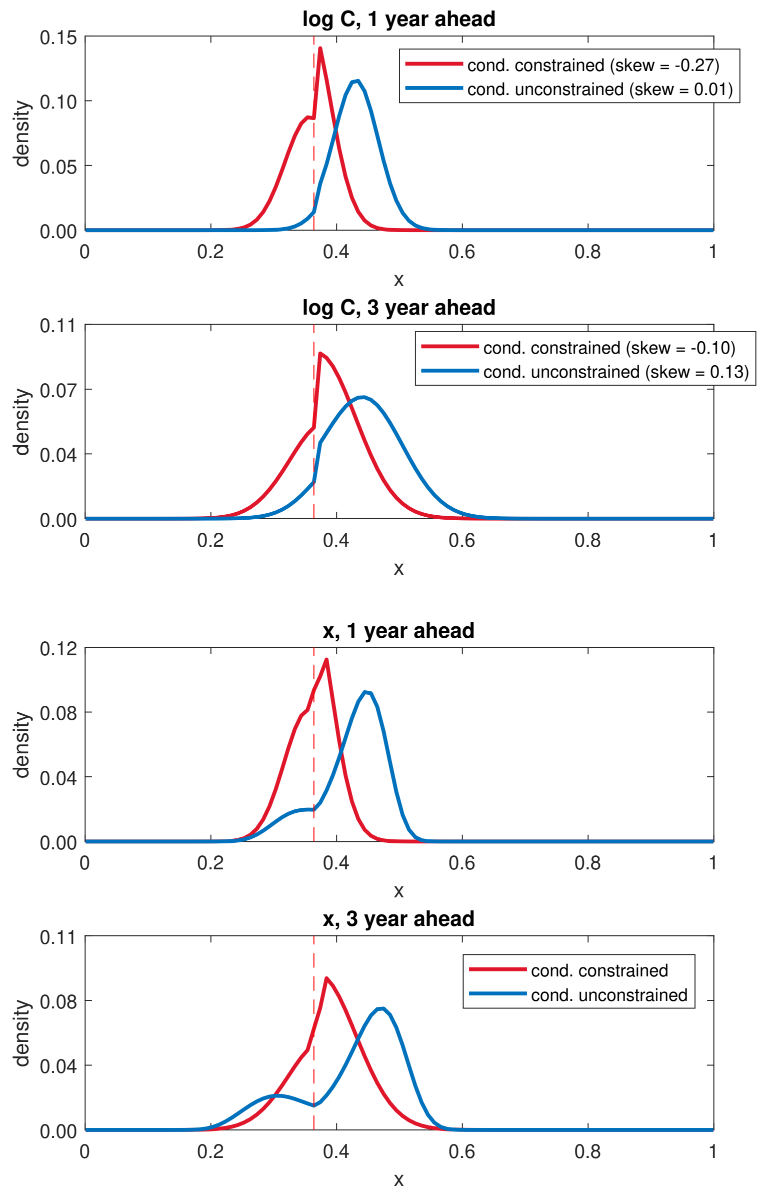
NOTES: This figure compares the model's solution for a lower risk aversion (γ=2) and a the baseline risk aversion (γ=5).

FIGURE 6. Slope and Credit



NOTES: This figure shows the slope of the yield curve and and the credit in the model. Credit is the position of financiers' short-term deposits as a share of total net worth.

FIGURE 7. Term structure of conditional distributions for real variables



NOTES: This figure shows the conditional distributions of consumption 1 and 3 years ahead (top two panels), and the conditional distributions of the endogenous state variable x 1 and 3 years ahead (bottom two panels). The distributions are conditional on the state of the economy being an unconstrained one (blue lines) or a constrained one (red lines). The red dashed line is the point in the state space at which constraints binds.

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6 Appendix

First, I derive the law of motion of the endogenous state variable, x_t, apply Ito's lemma to $\frac{n_{f,t}}{q_t}$:

$$dx_t = \frac{dn_{f,t}}{q_t} - \frac{n_{f,t}}{q_t^2} dq_t + \frac{n_{f,t}}{q_t} (dq_t)^2 - \frac{dq_t}{q_t} dn_{f,t} + \lambda \overline{x} q_t - \lambda n_{f,t}, \tag{9}$$

where the last term, $\lambda \overline{x} q_t - \lambda n_{f,t}$ represents the initial capital received by intermediaries, net of their aggregate dividends. Then, substituting the law of motion for q_t and $n_{f,t}$ in (9)

$$\begin{aligned} dx_t &= x_t \left[r_t + \frac{q_t \theta_{f,t}}{n_{f,t}} \left(E_t \left[dR_{f,t} \right] - r_t \right) - \mu_{q,t} + \left(\frac{q_t \theta_{f,t}}{n_{f,t}} - 1 \right) \sigma_{q,t}^2 \right] dt + \\ & \quad x_t \left(\frac{q_t \theta_{f,t}}{n_{f,t}} - 1 \right) \sigma_{q,t} dW_t + \lambda \left(\overline{x} - x_t \right) dt. \end{aligned}$$

Based on the previous expression, I denote

$$x_t \mu_{x,t} = x_t \left[r_t + \frac{q_t \theta_{f,t}}{n_{f,t}} \left(E_t \left[dR_{f,t} \right] - r_t \right) - \mu_{q,t} + \left(\frac{q_t \theta_{f,t}}{n_{f,t}} - 1 \right) \sigma_{q,t}^2 \right] + \lambda \left(\overline{x} - x_t \right) \tag{10}$$

$$x_t \sigma_{x,t} = x_t \left(\frac{q_t \theta_{f,t}}{n_{f,t}} - 1 \right) \sigma_{q,t} \tag{11}$$

Then, the system of ordinary differential equations in proposition 2 is characterized by using Ito's lemma and substituting the drift and diffusion of each function $p(x), \psi(x), \{P(x,\tau)\}_{\tau \geq 0}$. For example, for $p(x)$,

$$\begin{aligned} \mu_p(x) &= \frac{p'_x}{p} x \mu_x + \frac{1}{2} \frac{p''_{xx}}{p} (x \sigma_x)^2, \\ \sigma_p(x) &= \frac{p'_x}{p} x \sigma_x, \end{aligned}$$

where $x\mu_x$ and $x\sigma_x$ are from (10) and (11). Next, the system of ordinary differential equations consists of combining first order conditions with their corresponding value functions and market clearing conditions.

The recursive formulation of saver's problem is

$$0 = \max_{c_t,\theta_s} \frac{c^{1-\gamma}}{1-\gamma} - \rho J + E\left[dJ\right], \tag{12}$$

$$st.(1) \tag{13}$$

where $J(x, n_s)$ is the saver's value function. The first order conditions are for c and θ_s are

$$c^{-\gamma} = J'_{n_s} \tag{14}$$

$$J'_{n_s}\left(E\left[dR_s\right] - r\right) + J''_{n_s n_s} q\theta_s \sigma_q^2 + J''_{n_s x}\sigma_q\left(x\sigma_x\right) = 0 \tag{15}$$

where (15) holds with equality when $\theta_s > 0$. From (14)-(15) I obtain the expression for the real interest rate and the price q when $\theta_s > 0$, as follows. First, I define $m_t = e^{-\rho t} c_t^{-\gamma}$, which by the envelope condition (14) is $m_t = e^{-\rho t} V'_{n_s}$. Computing $\frac{dm_t}{m_t}$ yields

$$\frac{dm_t}{m_t} = -\rho dt + \frac{J''_{n_s n_s}}{J'_{n_s}} dn_{s,t} + \frac{1}{2}\frac{J'''_{n_s n_s n_s}}{J'_{n_s}}\left(dn_{s,t}\right)^2 + \frac{J''_{n_s x}}{J'_{n_s}} dx_t + \frac{1}{2}\frac{J'''_{n_s xx}}{J''_{n_s}}\left(dx_t\right)^2 + \frac{J'''_{n_s n_s x}}{J'_{n_s}}\left(dx\right)\left(dn_{s,t}\right) \tag{16}$$

Second, substituting (14)-(15) into the value function (12), I get

$$0 = \frac{\gamma}{1-\gamma}\left(J'_{n_s}\right)^{\frac{\gamma-1}{\gamma}} - \rho J + n_s r + J'_x E\left[dx\right] + \frac{1}{2} J''_{xx} E\left[dx^2\right]. \tag{17}$$

Third, take the derivative of (17) with respect to n_s, and then substract (16) to get the standard (Cox, Ingersoll and Ross (1985a))

$$E_t\left[\frac{dm_t}{m_t}\right] = -r_t dt$$

Lastly, using the diffusion of $\frac{dm}{m_t}$ in (16), we have the ordinary differenctial equation for q_t when $\theta_{s,t} > 0$,

$$E_t\left[dR_s\right] - r_t dt - E_t\left[\frac{dm_t}{m_t} dR_s\right] = 0,$$

that can be written as the forth equality of proposition 2.

I next turn into the financiers' optimality conditions and their value function. As with savers, I first start with the recursive formulation of financier's problem is

$$\begin{aligned} 0 &= \max_{\theta_f} \lambda\left(n_f - V_f\right) dt + \frac{1}{m} E\left[d\left(mV_f\right)\right] \\ & \quad st.(3)\ (4), \end{aligned} \tag{18}$$

where $V_f(x,n)$ is the financier's value function. As defined in the text, due to the linearity properties of the problem, $\psi(x) = V_f/n_f$.. The first order conditions, when $\theta_{f,t} > 0$, are

$$E_t\left[dR_s\right] - r_t dt - E_t\left[\left(\frac{dm_t}{m_t} + \frac{d\psi_t}{\psi_t}\right) dR_s\right] = 0. \tag{19}$$

Substituting (19) into the value function (18) yields the second and forth equation in proposition 2.

The last equations to derive are the princing formulas for the zero-coupon bonds (third and sixth equations in proposition 2). For this, I define the process π_t as the price of risk

$$\pi_t = \begin{cases} -\gamma\sigma_{c,t} & \text{if constraints binds} \\ -\gamma\sigma_{c,t} + \sigma_{\psi,t} & \text{if constraints is slack} \end{cases}$$

The price of the zero coupon bond is

$$P_t^{(\tau)} = P\left(x,\tau\right) = E_t\left[\int_t^\tau \frac{\widetilde{m}_s}{\widetilde{m}_t} ds\right], \tag{20}$$

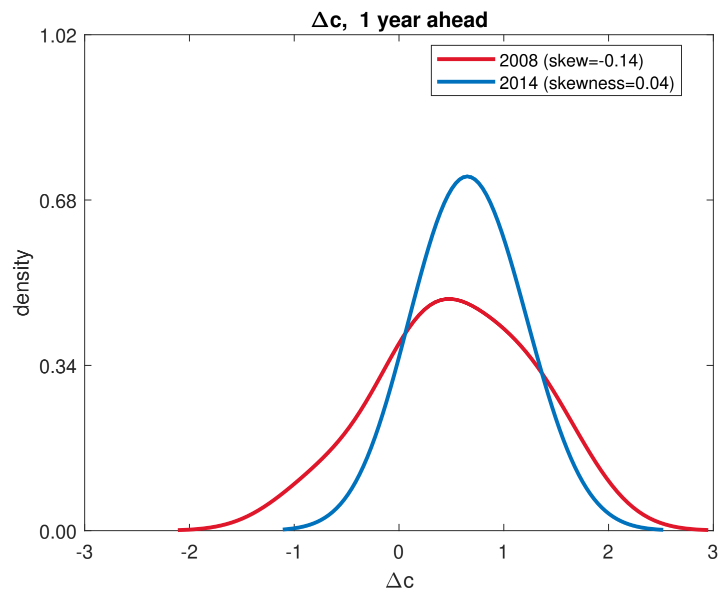
with $P\left(x,0\right) = 1$, and where $\widetilde{m}_t$ follows

$$\frac{dm_t}{m_t} = -r_t dt - \pi_t dW_t.$$

Then, solve (18) applying Feynman-Kac

$$\begin{aligned} -\frac{P'_\tau\left(x,\tau\right)}{P} + \frac{P'_x\left(x,\tau\right)}{P} x\mu_x + \frac{1}{2}\frac{P''_{xx}\left(x,\tau\right)}{P}\left(x\sigma_x\right)^2 - \pi\frac{P'_x\left(x,\tau\right)}{P} x\sigma_x &= 0 \\ P\left(x,0\right) &= 1 \,\forall x \end{aligned}$$

FIGURE 8. Conditional distributions of consumption growth.



NOTES: This figure shows the conditional distributions of consumption growth computed as in Adrian et al. (2019). That is, I run quantile regressions of consumption growth onto previous consumption growth and the National Financial Condition Index elaborated by the Federal Reserve Bank of Chicago. The red line shows the distribution of the 1-year-ahead consumption growth conditioning on tight financial conditions (2008Q3) and the blue line shows the the distribution of the 1-year-ahead consumption growth conditioning on loose financial conditions(2014Q1).