Financial Intermediaries and the Yield Curve

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Abstract

I study the yield curve dynamics in a general equilibrium model with financial intermediaries facing financing constraints. The economy features a positive real term premium in equilibrium stemming from financing constraints that occasionally bind. A flat yield curve reduces intermediaries' incentives to engage in maturity transformation and therefore is associated with lower levels of credit. I show that this mechanism 1) is a plausible reason for why a flattening of the yield curve precedes recessions and 2) also rationalizes why the term structure of distributions of future real outcomes are negatively skewed when financial conditions are tight.

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One of the main lessons of the large body of research on the nexus between financial intermediation and the macroeconomy is that financial intermediaries face constraints that distort the allocation of goods and capital—hence affecting agents' marginal valuations. In this paper, I argue that the yield curve contains information about such distortions because long-term yields are, by definition, a forecast of the economy's marginal valuation (Alvarez and Jermann (2005)). In other words, intermediaries' constraints are a macro source of term premia which means long-term yields and intermediaries' balance sheet dynamics are closely related.

For this, I study a canonical general equilibrium intermediary asset pricing model to underscore the mechanism through which intermediaries' constraints cause a positive term premium. I show that the connection between intermediaries' constraints, marginal valuations, and long-term yields help us in rationalizing the salient properties of the U.S. real and nominal yield curves. In particular, the model features highly nonlinear real yields, with an average upward-sloping real and nominal yield curves and highly volatile long-term yields, facts that have proven very difficult to match in representative agent models (Duffee (2018)). These results are purely driven by the fact that financial intermediaries face occasionally binding constraints. Indeed, if intermediaries were always unconstrained, then the yield curve would be flat and constant.

The mechanism is grounded in two main elements: intermediaries operate with leverage in equilibrium, and they face financing constraints. These two elements have been extensively studied in the macro-finance literature, but in this paper I focus the analysis on the yield curve.[1] To obtain leverage in equilibrium, I follow Brunnermeier and Sannikov (2014), among others, and I assume intermediaries are more efficient

[1] Recent literature, reviewed below, has departed from the representative agent analysis of the yield curve but without stressing the role of financing constraints—a salient characteristic of intermediaries.

in handling risky assets. That is, financial intermediaries issue short-term deposits to savers to fund positions in long-term risky assets and take advantage of their relatively better investment technology. However, intermediaries' positions in long-term risky assets can be constrained in certain states of the world due to agency problems, as in Gertler and Kiyotaki (2015). As a consequence, when intermediaries hit their constraints, they are forced to sell risky assets to less efficient savers and, subsequently, aggregate consumption and asset prices decline, the price of risk increases, and the wealth of financial intermediaries deteriorates even further, which force intermediaries to reallocate their portfolios, and so on. This well-known feedback mechanism has important implications for the yield curve, as I detail next.

The presence of occasionally binding constraints implies the economy features a bimodal distribution: It spends the vast majority of time in a "normal regime," in which constraints are slack, risk premia are low, the real interest rate is low, and volatility of asset prices is moderate. When negative aggregate shocks occur, the economy can enter in a "crisis regime" in which financing constraints are binding. Here, intermediaries reallocate their portfolios and wealth is transferred to inefficient savers. Savers' inefficiencies in handling risky assets causes deadweight losses and pushes the consumption level persistently below the trend growth and, therefore, the real interest rate persistently increases as agents perceive the "crisis regime" as transitory—the consumption level will recover its trend in the future. But this occurs precisely when the price of risk spikes, implying that real bond prices go down in states in which the marginal investor values those resources the most—a "crisis regime." Thus, real bonds carry an endogenously time-varying term premium and the yield curve is upward sloping, on average, due to the fact there is always a non-zero probability the economy can hit financing constraints.

I extend the analysis to study the nominal yield curve by introducing a simple monetary policy rule that is subject to persistent monetary shocks. The monetary policy rule, which takes the form of a Taylor rule, pins down an equilibrium inflation process that depends on the state of the economy as well as on persistent monetary policy shocks. The equilibrium nominal yield curve nominal is consistent with the empirical evidence on nominal yields as long as the reaction function of the monetary policy rule with respect to inflation is greater than one-by-one. As noted in Schneider (2022), if the model can capture the main properties of the real yield curve, then the nominal yield curve can be simply rationalized with an empirically plausible Taylor rule (i.e., a rule in which the monetary authority adjusts the interest rate more than one-by-one with inflation). I extend the analysis in Schneider (2022) to include persistent monetary policy shocks and therefore have a flexible environment in which the nominal yield curve is driven by both real and nominal shocks.

Besides accounting for the salient properties of the real and nominal yield curves (positive average term premium and highly volatile long-term yields), I show that the mechanism relating financial intermediary wealth and the yield curve rationalizes interesting macroeconomic phenomena. The purpose of these exercises is to illustrate that the mechanism in the model can rationalize evidence beyond the scope of the yield curve, therefore providing external validation of the key economic forces in the model.

First, there is ample reduced-form evidence indicating that a flattening of the yield curve—a smaller difference between long- and short-term rates—is associated with lower future economic activity. I show that the model contains a positive relationship between the slope of the yield curve and the quantity of credit intermediated in the economy. In the model, a flattening of the yield curve (caused by a reduction in the term premium instead of the expected path of short-term interest rates) reduces the intermediaries' incentives to engage in maturity transformation. I argue that this mechanism could be, at least partially, a plausible reason for why a flattening of the yield curve precedes recessions: A flattening of the yield curve is associated with lower credit growth—and, potentially, lower economic activity.[2]

Second, recent literature has stressed the role of financial conditions in driving the distribution of real variables in the near future (Adrian, Boyarchenko and Giannone (2019); Giglio, Kelly and Pruitt (2016)). More precisely, when financial conditions deteriorate, the forecasted conditional distribution of GDP growth becomes more negatively skewed. Moreover, this distribution changes with the forecast horizon; there is a term structure of conditional distributions that changes over the forecast horizon. The conditional distributions of real variables are intimately related with the yield curve, because long-term yields are conditional expectations of future variables (a point estimate), while the forecasted distribution includes computing the entire distribution of future realizations. To rationalize the evidence, I compute the term structure of the conditional probability density function of consumption and intermediaries' wealth across the horizon. This is the model's theoretical counterpart of the estimated conditional distributions in, for example, Adrian et al. (2019). I show the model captures the evidence relatively well: conditional on a state in which intermediaries are constrained (tight financial conditions), the term structure of conditional distributions of consumption exhibit a negative skewness. When financial conditions are loose, the negative skewness vanishes and the term structure of conditional distributions is roughly Gaussian.

[2] Minoiu, Schneider and Wei (2022) investigate in detail the connection between the term premium and banks' lending.

Related literature. This paper relates to a strand of literature that has departed from the representative agent analysis of the yield curve. In this line, part of the literature has stressed the role of certain agents (arbitrageurs, intermediaries, etc.) in explaining the yield curve dynamics, typically in a partial equilibrium setup (Vayanos and Vila (2021), Greenwood and Vayanos (2014), Haddad and Sraer (2019)). Relative to this literature, the contribution of this paper is to use a general equilibrium framework to study the role of financing constraints in driving the yield curve dynamics.[3]

The general equilibrium framework I build on (He and Krishnamurthy (2013), Brunnermeier and Sannikov (2014), Gertler and Kiyotaki (2015), among many others) has been extensively studied in the macro-finance literature to answer a variety of questions, particularly after the Great Recession. For example, Gertler and Karadi (2011) study unconventional monetary policies; Van der Ghote (2021) studies the coordination of conventional and macroprudential policies; Maggiori (2017) studies the risk sharing dynamics between countries that differ in their degree of financial development; Bigio and Schneider (2017) analyze the role of financing constraints and liquidity shocks in driving the equity premium. Relative to this literature, the contribution of this paper is to shift the focus away from equities (or "capital") to the yield curve dynamics. In particular, I show that financing constraints play a crucial role in producing an endogenously time-varying real term premium. Also, I show the connection between the yield curve and financial intermediaries' wealth is important for understanding why changes in the yield curve are associated with recessions, and also to understand why tight financing constraints imply a negatively skewed distribution of future economic outcomes.

[3]Other papers have studied the yield curve in a general equilibrium setup with heterogeneous agents (e.g., Wang (1996), Schneider (2022), Ehling, Gallmeyer, Heyerdahl-Larsen and Illeditsch (2018), among others) but without financing constraints.

1 Model

I propose a general equilibrium model with a financial intermediary sector along the lines of Brunnermeier and Sannikov (2014), He and Krishnamurthy (2013), and Gertler and Kiyotaki (2015) and focus on the pricing implications for the yield curve. I first solve for the real equilibrium and derive the real yield curve. I next extend the analysis to include a monetary policy rule and derive the nominal yield curve.

Time is continuous and denoted by $t > 0$. Aggregate output, denoted by Y_t, follows

$$\frac{\mathrm{d}Y_t}{Y_t} = \mu \mathrm{d}t + \sigma \mathrm{d}W_t,$$

where parameters $\mu > 0$, $\sigma > 0$ are constants and W_t is a standard Brownian motion in a complete probability space (Ω, F, P) .

The economy is populated by a continuum of savers (denoted by s) and a continuum of financiers (denoted by f). Financiers are in charge of managing financial firms, which are owned by savers, while the savers maximize their discounted utility from consumption. Both f and s are allowed to trade risky assets, but a key difference between f and s is that the former have a comparative advantage in operating risky assets over the latter—which implies f and s engage in borrowing and lending in equilibrium.[4]

Agents can trade two classes of assets—namely, long-term risky assets and short-term risk-free deposits. The long-term asset is in exogenous fixed supply and I denote its ex-dividend price by q_t. This asset pays a dividend Y_t per unit of time if held by f, but ωY_t, $\omega < 1$, if held by s. That is, it is more costly for savers than financiers to oper-

[4]Although my assumption about f and s having different expertise in handling the risky assets differs from He and Krishnamurthy (2013)—who assume segmented markets and that only "specialists" can trade risky assets—the asset pricing implications of both assumptions are similar (see Brunnermeier, Eisenbach and Sannikov (2013)).

ate this risky asset.[5] The purpose of this assumption is to obtain endogenous leverage in equilibrium, in the same way as in Brunnermeier and Sannikov (2014): Financiers have an advantage in handling risky assets and therefore will borrow from savers to take leveraged positions in risky capital. A direct consequence of assuming $\omega < 1$ is that when financiers' wealth is impaired and savers are handling risky assets, the aggregate dividend (and, in equilibrium, consumption) will decline.

The total return of investing in the long-term asset consists of the dividend yield plus the capital gains. For financiers, this is

$$\mathrm{d}R_{f,t} = \frac{Y_t}{q_t}\mathrm{d}t + \frac{\mathrm{d}q_t}{q_t},$$

while for savers the total return is

$$\mathrm{d}R_{s,t} = \frac{\omega Y_t}{q_t}\mathrm{d}t + \frac{\mathrm{d}q_t}{q_t},\ \omega < 1.$$

Second, the short-term deposit account is in zero net supply, and it yields a risk-free interest rate, denoted by r_t. For simplicity, I solve the model with the generic long-term asset q_t and the short-term deposit account. I introduce zero-coupon bonds of all maturities that are also in zero net supply below. That is, zero-coupons are redundant in the construction of the equilibrium, but they are useful to characterize the economy's equilibrium yield curve.

Savers consume and save. They have recursive preferences as in Duffie and Epstein

[5] This assumption about ω is equivalent to assume that savers have to pay a cost to operate risky assets (Gertler and Kiyotaki (2015))

(1992b) and their utility function is given by

$$U_t = \mathbb{E}_t \left[\int_t^{\infty} f(c_u, U_u) \, \mathrm{d}u \right],$$

where

$$f(c, U) = \frac{\rho}{1 - \frac{1}{\psi}} \left\{ \frac{c^{1-1/\psi}}{\left[(1-\gamma) U\right]^{\left(\gamma - \frac{1}{\psi}\right)/(1-\gamma)}} - (1-\gamma) U \right\}. \tag{1}$$

In (1), c is the savers' consumption, ρ is the time preference, ψ is the elasticity of intertemporal substitution (EIS), and γ is the risk aversion.

The savers' problem consists of choosing how much to consume and save in order to maximize their expected discounted utility. They can allocate their portfolios between risk-free deposits issued by financiers and can also hold risky assets. Their optimization problem can be written as

$$\max_{\{c_t, \theta_{s,t}\}} U_t,$$

subject to

$$\begin{aligned} \mathrm{d}n_{s,t} &= \left[n_{s,t} r_t - c_t + q_t \theta_{s,t} \left(\mathbb{E}_t \left[\mathrm{d}R_{s,t} \right] - r_t \right) + T_t \right] \mathrm{d}t + q_t \theta_{s,t} \sigma_{q,t} \mathrm{d}W_t, \qquad (2) \\ n_{s,t} &\geq 0, \end{aligned}$$

where $n_{s,t}$ is the savers' net worth, $\theta_{s,t}$ is the holding of the risky asset, and T_t the net transfers received from financiers' profits. Transfers are locally riskless because below I assume the dividend policy implemented by financiers is so.

Financiers are in charge of managing a financial intermediary firm. They operate this firm by issuing deposits to savers as well as using their own wealth, $n_{f,t}$, but they face financing constraints (described below). Their objective is to manage the financial intermediaries' portfolio and do not consume.[6] Instead, they paid dividends to savers. To avoid financiers growing out of their constraints, I follow Gertler and Kiyotaki (2015) and I assume a simple dividend policy in which dividends follow an exogenous Poisson process with intensity λ.[7] After paying dividends, financiers receive a fraction $\overline{x}$ of the economy's total wealth to re-start the financial firm. Financiers' problem is to maximize the value of the firm (i.e., the expected discounted value of firms' dividends)—that is,

$$V_{f,t} = \max_{\theta_{f,t}} \mathbb{E}_t \left[\int_t^\infty \frac{m_u}{m_t} \lambda e^{-\lambda(u-t)} n_{f,u} \mathrm{d}u \right], \tag{3}$$

subject to

$$\begin{aligned} \mathrm{d}n_{f,t} &= \left[r_t n_{f,t} + q_t \theta_{f,t} \left(\mathbb{E}_t \left[\mathrm{d}R_{f,t} \right] - r_t \right) \right] \mathrm{d}t + \theta_{f,t} q_t \sigma_{q,t} \mathrm{d}W_t, \qquad (4) \\ V_{f,t} &\geq \kappa \theta_{f,t} q_t, \qquad (5) \\ n_{f,t} &\geq 0, \end{aligned}$$

where m_t is savers' marginal utility, defined below, and $\theta_{f,t}$ is financiers' holdings of the risky asset. Financiers face a financing constraint, (5), that can be motivated with a standard agency problem. Specifically, I follow Gertler and Kiyotaki (2015) and assume the value of the financial intermediary firm has to be greater than a fraction of the assets

[6] The assumption that financiers do not consume is different than in Brunnermeier and Sannikov (2014). Assuming there is perfect consumption insurance between the savers and the financers simplifies the solution of the model and the analysis of the yield curve, because savers are in charge of pricing consumption across time.

[7] Recall that financiers have a technological advantage over savers, a force that pushes financiers to grow out of financing constraints.

the firm holds. This constraint operates as an endogenous leverage constraint.

I next define a competitive equilibrium.

Definition 1 (Competitive equilibrium) *A competitive equilibrium is a set of aggregate stochastic processes: prices* q_t, r_t, *policy functions for savers* $(\theta_{s,t}, c_t)$, *policy functions for financiers'* $\theta_{f,t}$, *the value of the financiers' firm* $V_{f,t}$, *and the utility of savers* U_t, *such that*

1. *Given prices,* $(\theta_{s,t}, c_t)$ *solves savers' problem*

2. *Given prices,* $\left(\theta_{f,t}, V_{f,t}\right)$ *solves financiers' problem*

3. *Markets clear (long-term asset, consumption good, and short-term debt)*

$$\begin{aligned} \theta_{s,t} + \theta_{f,t} &= 1, \\ c_t &= \omega \theta_{s,t} Y_t + \theta_{f,t} Y_t, \\ n_{f,t} + n_{s,t} &= q_t, \end{aligned}$$

where the last equation (market clearing for short-term debt) is redundant due to Walras' Law, but is useful to explicitly show that wealth holdings add up to total wealth q_t.

The market clearing condition for the goods market, which shows that consumption must be equal to the aggregate dividends, is crucial for understanding the results. When savers hold risky assets, $\theta_{s,t} > 0$, the aggregate consumption falls because there are deadweight losses associated with savers handling risky assets. One interpretation of the assumption about ω is that financiers lend resources to firms that are more productive than savers when producing consumption goods, as in Brunnermeier and Sannikov (2014). An alternative interpretation is that savers need to pay a cost when holding risky assets, which captures savers' lack of expertise relative to financiers in screening and monitoring investment projects, as in Gertler and Kiyotaki (2015). I provide further discussion in the appendix about the quantitative implications of $\omega < 1$ for consumption and the price of risky capital, and I contrast those implications with the evidence.

Before turning to the solution of the model, it is useful to characterize agents' optimization problems with their first order conditions. For savers,

$$r_t = -\mathbb{E}_t \left[\frac{\mathrm{d}m_t}{m_t} \right],$$

and

$$\mathbb{E}_t \left[\mathrm{d}R_{s,t} \right] - r_t \mathrm{d}t \leq -\mathbb{E}_t \left[\frac{\mathrm{d}m_t}{m_t} \mathrm{d}R_{s,t} \right],$$

with equality if households are holding long-term assets (i.e., $\theta_{s,t} > 0$). The savers' stochastic discount factor, as noted in Duffie and Epstein (1992b), is given by

$$\frac{\mathrm{d}m_t}{m_t} = \frac{\mathrm{d}f_{c,t}}{f_{c,t}} + f_{U,t}\mathrm{d}t,$$

where f_c and f_U are the partial derivative of the aggregator (1) with respect to c and U, respectively.

The optimality conditions for financiers require a few more steps. First, notice because financiers' objective function and constraints are linear in wealth, the value function can be written as

$$V_{f,t} = \phi_t n_{f,t}, \tag{6}$$

where $\phi_t \geq 1$ can be interpreted as financiers' marginal value of wealth (as well as a

"Tobin's q").[8] Notice that ϕ_t is an endogenous Itô process whose drift $\mu_{\phi,t}$ and diffusion $\sigma_{\phi,t}$ are solved in equilibrium. Then the financing constraint can be written as

$$\begin{aligned} \phi_t n_{f,t} &\geq \kappa \theta_{f,t} q_t, \\ \phi_t &\geq \kappa \frac{\theta_{f,t} q_t}{n_{f,t}} \equiv \kappa \alpha_{f,t}, \end{aligned}$$

where $\alpha_{f,t}$ is the endogenous financiers' portfolio share in the risky asset. The financiers' problem can be written as

$$0 = \max_{\theta_{f,t}} \lambda \left(n_{f,t} - V_{f,t}\right) m_t \mathrm{d}t + \mathbb{E}_t \left[\mathrm{d}\left(m_t V_{f,t}\right)\right] + \chi_t \left(V_{f,t} - \kappa \theta_{f,t} q_t\right) \mathrm{d}t, \tag{7}$$

where χ_t is the Lagrange multiplier associated with the financing constraint. Using (6) and (7), the first order conditions for financiers can be written as

$$\mathbb{E}_t \left[\mathrm{d}R_{f,t}\right] - r_t \mathrm{d}t \geq -\mathbb{E}_t \left[\left(\frac{\mathrm{d}m_t}{m_t} + \frac{\mathrm{d}\phi_t}{\phi_t}\right) \mathrm{d}R_{f,t}\right],$$

with equality if $\chi_t = 0$. Put differently, financiers are the marginal investors in long-term risky assets if their constraints are not binding. If financing constraints are binding, then their holdings in risky assets are pinned down by such constraints (i.e., $\phi_t = \kappa \alpha_{f,t}$), and savers are the marginal investors in risky assets.

Real Yield Curve. I next characterize the yield curve in the economy, which consists of the endogenous price vector for real bonds denoted by $\left\{P_t^{(\tau)}\right\}_{\tau \geq 0}$, where τ represents the time to maturity of the bond. Yields can then be obtained simply as $y_t^{(\tau)} = -\log P_t^{(\tau)} / \tau$. I assume that the saver is the marginal investor in long-term zero-coupon

[8] See Gertler and Kiyotaki (2015).

real bonds. Intuitively, the saver is the marginal investor in the risk-free deposit, which is the relative price of a unit of consumption in the present versus the next instant. Hence, it is natural to assume the saver is also the marginal agent when pricing long-term bonds, which are the relative price of a unit of consumption in the present versus the near future. In addition, assuming the saver is the marginal investor for long-term bonds is consistent with the evidence documented in Haddad and Muir (2021), who show that Treasury bonds are one of the least intermediated asset class—suggesting financiers' are not marginal agents in such a market. The real bond price with time to maturity τ is given by

$$P_t^{(\tau)} = P(x, \tau) = E_t\left[\int_t^{t+\tau} \frac{m_u}{m_t} \mathrm{d}u\right],$$

which can be written as

$$\mathbb{E}_t\left[\frac{\mathrm{d}P_t^{(\tau)}}{P_t^{(\tau)}}\right] - r_t \mathrm{d}t = -cov_t\left(\frac{\mathrm{d}m_t}{m_t}, \frac{\mathrm{d}P_t^{(\tau)}}{P_t^{(\tau)}}\right).$$

I next extend the analysis to study the nominal yield curve.

Nominal yield curve. To compute nominal bond prices, I need to introduce money in the analysis. For this, I follow an extensive literature in term structure analysis in endowment economies and assume money is simply a unit of account (i.e., the consumption good is quoted in terms of money) that does not affect the real allocation (see Cox, Ingersoll and Ross (1985b), among others). More precisely, I assume that the inflation rate is pinned down endogenously by a monetary authority that follows and interest rate rule subject to monetary policy shocks. In short, the setup I propose is akin to a two-equation New Keynesian model, because I assume there is no Phillips curve.[9] For this, I

[9] See Galí (2015), chapter 2.

first define the nominal stochastic discount factor as

$$m_t^{\$} = \frac{m_t}{p_t}, \tag{8}$$

where p_t is the price level (i.e., the price of one unit of consumption good in terms of money). I assume the price level fluctuates smoothly (i.e., is not affected by Brownian shocks),[10]

$$\frac{\mathrm{d}p_t}{p_t} = \pi_t \mathrm{d}t,$$

so that π_t is the inflation rate. I assume the price level dynamics are pinned down by a central bank following a Taylor Rule. That is, the central bank sets a nominal interest rate i_t^{CB} as

$$i_t^{CB} = \delta_0 + \delta_\pi(\pi_t - \overline{\pi}) + s_t, \tag{9}$$

where s_t is a persistent random variable capturing monetary policy surprises, δ_π is the so-called Taylor loading on inflation, and $\overline{\pi}$ is an inflation target (which can be absorbed by the constant δ_0). The monetary policy surprises follow

$$\mathrm{d}s_t = -\lambda_s s_t \mathrm{d}t + \sigma_s \mathrm{d}W_{s,t},$$

where $W_{s,t}$ are monetary policy shocks uncorrelated with the endowment shock. Then, in equilibrium, the i_t^{CB} must be equal to the nominal interest rate priced by savers

$$i_t^{CB}\mathrm{d}t = -\mathbb{E}_t\left[\frac{\mathrm{d}m_t^{\$}}{m_t^{\$}}\right]. \tag{10}$$

[10]Implicitly, the assumption is that the monetary authority has the tools to pin down a smooth price level; see Di Tella and Kurlat (2021).

Using Itôs Lemma on (8) and replacing in (10), I obtain an endogenous inflation process

$$\pi_t = \frac{r_t - s_t - \delta_0}{\delta_\pi - 1}.$$

Then, the price of a nominal bond with time to maturity τ is

$$P_t^{\$,(\tau)} = P^{\$}(x, s, \tau) = \mathbb{E}_t \left[\int_t^{t+\tau} \frac{m_u^{\$}}{m_t^{\$}} \mathrm{d}u \right],$$

2 Model Solution

I use the homogeneity property of objective functions and constraints to solve the equilibrium in a recursive fashion, using a single endogenous state variable,

$$x_t = \frac{n_{f,t}}{q_t} \in [0, 1]. \tag{11}$$

The endogenous state variable x_t follows an Itô process with drift $\mu_{x,t}$ and diffusion $\sigma_{x,t}$, that I define in proposition 2 below. The solution of the model consists of solving the endogenous variables in a Markov equilibrium in x_t. For this, I characterize the equilibrium using the optimality conditions for savers and financiers as well as the market clearing condition. The proposition below shows the system of ordinary differential equations (ODEs) that solves for financiers' marginal value, $\phi(x_t)$, the price-dividend ratio $p(x_t) = q_t/Y_t$, the savers' value function $U(x, c)$, and the real and nominal bond prices.

Proposition 2 *The Markov equilibrium is characterized by the following system of ODEs. In the unconstrained region (i.e., when $x > x^*$ where x^* is such that $\forall$, $\frac{\phi(x)}{\kappa} > \alpha(x)$),*

$$
\begin{aligned}
0 &= \frac{1}{p(x)} + \mu_p(x) + \mu + \sigma_p(x)\sigma - r(x) + \left(\sigma_\phi(x) - \gamma\sigma_c(x) + \left(\frac{1}{\psi} - \gamma\right)\sigma_\xi(x)\right)(\sigma_p(x) + \sigma) \\
0 &= \frac{\lambda(1 - \phi(x))}{\phi(x)} + \mu_\phi(x) - \left(\gamma\sigma_c(x) + \left(\frac{1}{\psi} - \gamma\right)\sigma_\xi(x)\right)\sigma_\phi(x) \\
0 &= \frac{\rho}{1 - \frac{1}{\psi}}\left\{\xi(x)^{\frac{1}{\psi} - 1} - 1\right\} + \mu - \frac{1}{2}\sigma^2 + \mu_\xi(x) - \frac{\gamma}{2}\sigma_\xi(x)^2 + (1 - \gamma)\sigma_\xi(x)\sigma
\end{aligned}
$$

In the constrained region (i.e., when $x \leq x^*$ where x^* is such that $\forall$, $\frac{\phi(x)}{\kappa} > \alpha(x)$ $x \leq x^*$),

$$
\begin{aligned}
0 &= \frac{\omega}{p(x)} + \mu_p(x) + \mu + \sigma_p(x)\sigma - r(x) + \left(\left(\frac{1}{\psi} - \gamma\right)\sigma_\xi(x) - \gamma\sigma_c(x)\right)(\sigma_p(x) + \sigma), \\
0 &= \frac{\lambda(1 - \phi(x))}{\phi(x)} + \frac{\phi(x)}{\kappa}\left(\frac{(1 - \omega)}{p(x)} + \sigma_\phi(x)(\sigma_p(x) + \sigma)\right) + \mu_\phi(x) - \left(\gamma\sigma_c(x) + \left(\frac{1}{\psi} - \gamma\right)\sigma_\xi(x)\right)\sigma_\phi(x), \\
0 &= \frac{\rho}{1 - \frac{1}{\psi}}\left\{\xi(x)^{\frac{1}{\psi} - 1} - 1\right\} + \mu_c(x) - \frac{1}{2}\sigma_c(x)^2 + \mu_\xi(x) - \frac{\gamma}{2}\sigma_\xi(x)^2 + (1 - \gamma)\sigma_\xi\sigma_c(x).
\end{aligned}
$$

The real interest rate, $r_t = r(x_t)$, is $r_t = -\mathbb{E}_t\left[\frac{dm}{m}\right]$, where $m_t = \exp\left(\int_0^t f_U du\right) f_c$, real bonds are $P(x, \tau) = \mathbb{E}_t\left[\int_t^\tau \frac{m_u}{m_t} du\right]$, and nominal bonds are $P^\$(x, \tau) = \mathbb{E}_t\left[\int_t^\tau \frac{m_u^\$}{m_t^\$} du\right]$, solving

$$
\begin{aligned}
0 &= -\frac{P_\tau(x, \tau)}{P(x, \tau)} + \mu_P(x, \tau) + \frac{1}{2}\sigma_P(x, \tau)^2 - r(x) - \left(\gamma\sigma_c(x) + \left(\frac{1}{\psi} - \gamma\right)\sigma_\xi(x)\right)\sigma_P(x, \tau), \\
0 &= -\frac{P_\tau^\$(x, s, \tau)}{P^\$(x, s, \tau)} + \mu_P^\$(x, s, \tau) + \frac{1}{2}\sigma_P^\$(x, s, \tau)^2 - i(x, s) - \left(\gamma\sigma_c(x) + \left(\frac{1}{\psi} - \gamma\right)\sigma_\xi(x)\right)\sigma_{P^\$}(x, s, \tau),
\end{aligned}
$$

with $P(x, 0) = 1 \; \forall x$ and $P^\$(x, s, 0) = 1$, $\forall (x, s)$. The terms $\mu_p(x)$, $\mu_\phi(x)$, $\mu_\xi(x)$, $\mu_c(x)$, $\mu_P(x, \tau)$, $\mu_P^\$(x, s, \tau)$ and $\sigma_p(x)$, $\sigma_\phi(x)$, $\sigma_\xi(x)$, $\sigma_c(x)$, $\sigma_P(x, \tau)$, $\sigma_{P^\$}(x, s, \tau)$ are partial derivatives obtained applying Itô's lemma in their corresponding functions.

Proof. See appendix.

3 Results

Calibration. I calibrate the model at an annual frequency and solve it using global solution technique based on projection methods. I provide a detailed description of the solution method in the appendix. Table 1 shows the three groups of parameters—namely, Preferences and Endowment, Financiers, and Nominal rate. First, for Preferences and Endowment, as I highlight below, the risk aversion (γ) and the EIS ($1/\psi$) play a critical role in pinning down the dynamics of the real interest rate and the yield curve. This is because γ and ψ control the relative strength of the precautionary savings (i.e., changes in the interest rate followed by changes in volatility of consumption) and intertemporal smoothing (i.e., the changes in the interest rate caused by changes in expected change of consumption) forces. In the baseline calibration, I set γ=5 and $\psi = 1/5$, which means the savers have CRRA preferences. I later relax this assumption and solve the model with alternative preference parametrization and explain why the EIS play a critical role in the results. For the endowment, I set σ=0.036, a number that is in line with the volatility of productivity in the US and close to the value used in He and Krishnamurthy (2019).[11] Lastly, I set μ=0.007 to match the average level of the real rate.

Second, for financiers' technology and constraint, I calibrate κ=0.4 to target an average leverage of 3 (He and Krishnamurthy (2019)). I set λ=0.08, which gives an expected payout rate of the intermediary as in Gertler and Kiyotaki (2015), and I set $\overline{x} = 0.2$ to stabilize the wealth of intermediaries below 0.5. Lastly, ω is a crucial assumption, and I set ω=0.85. From a quantitative point of view, ω=0.85 imply that the price-dividend ratio and consumption can drop, at most, 50% and 15%, respectively (this would be the case

[11] Previous papers have used a wide range of values for σ. For example, He and Krishnamurthy (2013), in a similar setup, use σ=0.09; Brunnermeier and Sannikov (2014), also in a similar setup but with endogenous production, use σ=0.1.

when intermediaries have no wealth and savers hold the entire wealth in the economy). Importantly, in equilibrium, households will almost surely never hold the entire wealth in the economy (i.e., the probability of x reaching zero is almost surely zero). Indeed, only in very rare occasions the price-dividend ratio and consumption will drop more than 25% and 7%, respectively, in a given year.[12]

Finally, for the monetary policy rule, I set $\delta_\pi = 1.5$, which is a commonly used parametrization since Taylor (1993) and broadly consistent with the evidence. I calibrate the monetary policy shock to match the persistence and volatility of the surprises documented in Gertler and Karadi (2015).

Solution and Mechanism. Figure 1 shows the solution of the key endogenous variables. All figures display the endogenous variables in the Markov equilibrium (i.e., endogenous variables as a function of the state variable x). The red dashed line in all panels represents the point at which the financing constraint binds.

The invariant distribution, displayed in the lower-left panel of Figure 1, shows the economy has two modes. It spends the majority of the time in a normal regime where constraints are slack (i.e., to the right of the red dashed line), and some time in a crisis regime, where constraints are binding. Normal times are characterized by low volatility, low rates, and moderate leverage. As is common in these types of models, leverage is counter-cyclical: The lower the intermediaries' wealth (i.e., lower x), the higher the leverage.

If the economy is in the normal regime and a sufficiently negative aggregate shock occurs, financial intermediaries reallocate their portfolios, the price of risky assets de-

[12] I discuss further details about ω in the appendix and show that the equilibrium-implied losses in consumption and asset prices are broadly in line with the empirical evidence documented in Muir (2017) and Greenwood, Hanson, Shleifer and Sørensen (2022).

clines, and the price of risk increases. Financing constraints may bind (depending on the magnitude of the shock) and trigger the well-known financial accelerator mechanism studied in previous literature (e.g., Bernanke, Gertler and Gilchrist (1999)), in which lower valuations deteriorate intermediaries' wealth even further.

A central element in the yield curve dynamics is the behavior of the short-term interest rate, r, shown in the upper-left panel. Notice that when the economy enters a crisis regime, the price of risk spikes and the real interest rate increases. This is because wealth is transferred to savers, who are inefficient in handling risky assets, which means the level of aggregate dividends (and consumption) declines. Because the inefficiencies caused by the misallocation of risky assets are temporary, savers expect the consumption level to increase in the future, which causes an increase the real interest rate. Put differently, the dynamics for the consumption level are similar to a random-walk with drift, where deviations from the trend are persistent. When consumption is below the trend, it is expected to mean-revert in the future. In the model, the trend is endogenously driven by financial intermediaries' wealth dynamics.

The Real Yield Curve. Intuitively, investors require a premium to hold an asset whose value persistently declines in states in which the price of risk is high. This is precisely what drives the real term premium in the economy: real bond prices decline (i.e., real rate persistently increases) in states in which the price of risk is high. Figure 2 shows the average yield curve in the economy. Simply put, in the stochastic steady state—where expected short rates are constant—long-term yields are driven by the term premium, causing the yield curve to be upward sloping on average. The left panel of Figure 2 illustrates the dynamics of yields at different horizons across the state space. The mechanism through which financial intermediaries reduce their positions in risky assets by selling those to less efficient savers is more pronounced in short-maturity rates—long-term yields are less sensitive to the misallocation of wealth in the economy. Put differently, current fluctuations in financiers' wealth have a lower incidence in driving longer-maturity bonds, a feature that can be appreciated in the left panel of Figure 2. The panel shows the yield of bonds at 1-, 10-, and 30-year maturities and also displays the yield of a very long-term bond. As the horizon of the bond increases, the yields become less sensitive to current financial conditions: x_t has a smaller impact on yields' dynamics. This result, driven by the persistence and stationarity of x, shows that even very long-term rates can display substantial volatility.

The upper panel of figure 3 shows the real yield curve for different levels of x. The circles in the figure represent the average real yields.[13] The yield curve is flat when x is high mainly because term premiums and real rates are low. When x is high, intermediaries are relatively well capitalized and financing constraints are slack. When x is low, however, the economy is in crisis times, constraints are binding, and real yields are high. In this state, the short-term rate is expected to mean revert, and this force pushes down long-term rates (even though term premia is high, the expectations of the short-rate dominate). Thus, the gray line shows a downward-sloping yield curve. Finally, the lower panel of 3 shows the standard deviations of yields across maturity. As can be seen, the model can capture well the first and second moments of the real yield curve.

The Nominal Yield Curve. Figure 4 shows the nominal yield curve in the baseline calibration. The top panel displays the nominal yield curve when x is at its mean value and shows the yield curve at different values for the other state variable, the monetary policy shock s_t. In equilibrium, a low s_t translates into a higher inflation, which ultimately

[13] I use data for TIPS in the period 2002:Q1 to 2018:Q4 and Chernov and Mueller (2012) for 1971:Q3 to 2001:Q4. I explain in the appendix all the details about the data used in the paper.

causes higher nominal interest rates. This can be seen in the equilibrium nominal short rate, which is the result of plugging the endogenous inflation process (1) into the Taylor rule (9)

$$i_t = \widetilde{\delta_0} + \left(\frac{\delta_\pi}{\delta_\pi - 1}\right) r_t + \left(\frac{\delta_\pi}{1 - \delta_\pi}\right) s_t, \tag{12}$$

where $\widetilde{\delta_0}$ is an adjusted constant similar to δ_0. Then, as long as $\delta_\pi > 1$ (which is the empirically and theoretically relevant case), a higher (lower) s_t will, in equilibrium, cause lower (higher) nominal rates because of the endogenous response in inflation. The lower panel of Figure 4 shows the nominal yield curve for different values of x, holding s_t at its unconditional mean. As noted in Schneider (2022), $\delta_\pi > 1$ produces a nominal term premium that is larger than the real term premium. Hence, the model can capture the evidence that the slope of the nominal yield curve is, on average, approximately twice as big as the slope of the real yield curve. Intuitively, this is because, in order to pin down inflation, the monetary authority must adjust the nominal interest rate to changes in the state of the economy in a relatively stronger fashion than the adjustment of the real rate.

Time-varying Term Premium and Bond Return Predictability. A central property of the model is that expected excess returns on bonds are time-varying. As a consequence, long-term yields fluctuate not only because of changes in the expected path of short-term rates, but also due to changes in term premium. In other words, the so-called expectations hypothesis is rejected in the model, a feature that is consistent with a large body of evidence.[14]

The upper-left panel in Figure (5) shows the dynamics of the term premium in the model, for different maturities, across the state space. The average term premium is

[14] Duffee (2013) provides a summary.

positive and increasing across maturities, as shown by the upper-right panel of Figure (5). Intuitively, bonds of longer maturity contain more interest rate risk than bonds of short maturity, and, as a consequence, carry a larger premium. Notice that the term premium spikes when the constraints bind, as the price of risk increases and real bond prices decline, in a similar dynamic to the one explained in the mechanism in the previous subsection.

Because the term premium fluctuates over time, a natural question to ask is whether information in the yield curve at time t helps in predicting future fluctuations in time term premia. In particular, the seminal work of Fama and Bliss (1987) shows that the forward-spot spread predicts future excess returns on bonds, which is one of the salient properties of the empirical evidence about the yield curve. I next study the extent to which the predictability of bond returns is captured by the model. For this, I conduct a predictability analysis following Fama and Bliss (1987). I run, using simulated data from the model, the following regressions

$$rx_{t+1}^{(\tau)} = \alpha^{(\tau)} + \beta^{(\tau)} \left(f_t^{(\tau)} - y_t^{(1)} \right) + \epsilon_{t+1} \tag{13}$$

where $rx_{t+1}^{(\tau)} = p_{t+1}^{(\tau)} - p_t^{(\tau)} - y_t^{(1)}$, with $p_t^{(\tau)} = \log P_t^{(\tau)}$, are the excess returns of a $\tau-$maturity bond; $f_t^{(\tau)}$ is the one year forward between maturity n and $n-1$; and $y_t^{(1)}$ is the one-year rate.

As noted in Fama and Bliss (1987), a positive $\beta^{(\tau)}$ indicates that term premium fluctuates through time and that a higher forward-spot spread predicts higher expected excess returns on bonds. Figure (5) shows the results for the $\beta^{(\tau)}$ in the model, for maturities between 2 and 5 years (as in Fama and Bliss (1987)). As shown, model-implied coefficients are positive across all maturities, which implies the forward-spot spread in the model contains information about future bond excess returns, consistent with the evidence reported in Fama and Bliss (1987).

A complementary way to illustrate the predictability results in the model is to numerically compute the elements in regression (13) and study how they change across the state space. For this, I study how expected excess return—the conditional expectations of the regression (13)—and the forward-spot spread $f_t^{(n)} - y_t^{(1)}$—the right-hand side of the regression (13)—change across the state space.

The lower-right panel of Figure (5) shows the expected excess returns for a 5-year bond as well as the 5-year forward-spot spread across the state space. Consistent with the estimates of equation (13) shown in the lower-left panel of Figure (5), as well as the results in Fama and Bliss (1987), the model implies a positive co-movement between the expected excess returns and the forward-spot spread. In other words, movements in long-term one-year forwards (relative to the one spot one year rate) is primarily driven by the spike in term premium.

The Role of Risk Aversion and the EIS. The risk aversion and EIS parameters play a crucial role in characterizing how the real interest rate fluctuates across the state space. Figure 6 shows the slope of the real yield curve (left panel) and the real interest rate (right panel) for three different calibrations: the baseline calibration ($\gamma = 5$ and $\psi = 1/5$); a calibration with a higher EIS but the same risk aversion than the baseline ($\gamma = 5$ and $\psi = 1/3$); and a calibration with a higher EIS and lower risk aversion than the baseline ($\gamma = 3$ and $\psi = 1/3$). The purpose of these three calibrations is to disentangle the role of the EIS and the risk aversion in driving yields' dynamics.

The baseline calibration is shown in blue in both panels, and it's useful to use it as a reference to compare against the alternative calibrations. The red bar on the left panel and the red-dashed line on the right panel show the slope of the real yield curve and the real interest rate, respectively, for a calibration with higher EIS but the same risk aversion as the baseline ($\gamma = 5$ and $\psi = 1/3$). As can be seen, increasing the EIS —while holding the risk aversion constant—reduces the term spread to approximately one-third of the term spread obtained in the baseline calibration. The reduction in the term spread is because a higher EIS increases the relative importance of the precautionary savings channel over the intertermporal smoothing channel: When the constraint binds and the volatility of consumption increases, the real interest rate declines, as shown in the right panel. The decline in the real interest rate implies that real bond prices increase in bad times, hence pushing down the real term premium. As the wealth of intermediaries decline further, the pricing impact of intermediaries decline so that volatility decreases. Then, the intertemporal smoothing channel dominates again, and the real rate increases as the consumption declines further with lower x.

The gray bar on the left panel and gray dotted line on the right panel shows the term spread and real rate in the case of lower risk aversion and higher EIS than the baseline calibration. The results are similar to the case in which only the EIS is higher than the baseline. The real interest rate declines somewhat when the financing constraint binds, which indicates that the precautionary savings is stronger than the intertemporal smoothing force but less so than in the case with ($\gamma = 5, \psi = 1/3$). Importantly, notice that the case with ($\gamma = 3, \psi = 1/3$) implies similar term spread than the case with the ($\gamma = 5, \psi = 1/3$). This indicates that the vast majority of the decline in the term spread from the baseline calibration to the CRRA case with lower risk aversion of $\gamma = 3$ can be attributed to the EIS rather than the risk aversion coefficient.

Credit Cycle and the Yield Curve. A relatively well-known empirical regularity is that the slope of the yield curve is associated with changes in future economic growth. One possible explanation for the connection between the slope of the yield curve and future GDP growth is that the slope of the yield curve captures investors' expectations about the future path of the short-term interest rate. Hence, a reduction in the slope of the yield curve could be attributed to investors expecting lower rates down the road as the recession unfolds. Under this explanation, the slope of the yield curve and GDP growth are linked through investors' expectations about the path of the short-term interest rate.

An alternative explanation, not mutually exclusive with the previous one, could be that the slope of the yield curve changes due to the term premium instead of the expected path of future rates. The model presented in Section 2 offers a plausible explanation for why changes in the slope (driven by changes in the term premium) may ultimately affect GDP, because it links yields with the amount of intermediated credit by financiers. Although the model does not include an explicit production sector, a basic extension can be added to link credit and production.[15] Hence, through the lens of the model's mechanism, a plausible explanation for how the yield curve may cause changes in GDP growth is that the yield curve affects the supply of credit. In other words, a lower term spread, driven by changes in the term premium instead of the expected path of short rates, reduces financiers' incentives to engage in maturity transformation, hence lowering the aggregate amount of credit in the economy.

Figure 7 shows the model's prediction for the relationship between credit and the slope of the yield curve. The positive correlation between the slope and credit indicates that states in which the term spread is large correspond to expansions in credit. An intuitive explanation for the positive correlation between the slope of the yield curve

[15] Many papers have studied the connection between aggregate credit and the business cycle. See Bernanke and Gertler (1989), among others.

and aggregate credit is that a larger term spread makes it more profitable to engage in maturity transformation and therefore financiers expand credit. However, by definition, the slope of the yield curve fluctuates due to both changes in the expected path of the short rate as well as changes in the term premium. Only the latter, changes in the term premium, affects intermediaries' willingness to engage in maturity transformation. I next show that the positive correlation between aggregate credit and the slope of the yield curve in the model is driven by changes in the term premium instead of changes in the expected path of short rates.

The yellow dashed line in figure 7 shows the slope of the yield curve assuming the local expectation hypothesis (LEH) is valid.[16] The LEH consists essentially in computing the yield curve as if the term premium was zero and constant. The yield curve under LEH is computed as

$$y_t^{e,(\tau)} = -\frac{1}{\tau} \log E_t \left[\exp \left(- \int_t^{t+\tau} r_u \mathrm{d}u \right) \right], \tag{14}$$

where $y_t^{e,(\tau)}$ is the average expected yield between t and $t + \tau$. Then, the slope under the LEH is just the spread across the yields computed in 14. Notice that the slope of the yield curve under LEH is slightly negative and declines as the level of rates increases, and it is therefore almost uncorrelated with changes in aggregate credit.[17] As a consequence, the dynamics of term premia is the main reason for why the slope of the yield curve co-moves positively with the aggregate amount of credit.

To test the positive relationship between the slope of the yield curve and credit, I

[16] Cox, Ingersoll and Ross (1981) prove that the LEH is the version of the expectation hypothesis that is consistent with a continuous-time, rational expectations equilibrium.

[17] The slope under LEH is slightly negative due to the presence of a Jensen's inequality term. See Piazzesi (2010), Section 2, for a detailed treatment.

run the following regression:[18]

$$\Delta credit_t = \alpha + \beta \Delta slope_{t-1} + \gamma' \boldsymbol{\kappa_{t-1}} + \varepsilon_t. \tag{15}$$

In 15, $\Delta credit_t$ is the log-difference of real total loans to non-financial private sector between t and $t-1$, $\Delta slope_{t-1}$ is the difference between long and short interest rates in $t-1$, and $\boldsymbol{\kappa_{t-1}}$ are controls.[19] The source of the data is Jordà, Schularick and Taylor (2016), and I run the regressions in two subsamples, the full sample (1870-2016) and the postwar sample (1946-2016).

Table 2 shows the results.[20] The slope of the yield curve is associated with the evolution of total credit in the economy. The positive association between credit and the slope of the yield curve is robust across subsamples and also after controlling for GDP growth or changes in the slope. Through the lens of the model, the positive association between credit and the slope of the yield curve is due to the presence of financial intermediaries: A steeper (flatter) yield curve incentive intermediaries to expand (contract) credit.

Term structure of conditional distributions. Recent literature has stressed the role of financing conditions when forecasting the distribution of future real variables (Adrian et al. (2019)). The term structure of distributions of real variables is related to the yield curve because the former is a forecast of the conditional distribution of a random variable —the marginal utility in the economy—at a certain point in the future, while the

[18] Minoiu et al. (2022) provide more granular evidence about the empirical relationship between term premium and banks' lending.

[19] I take the difference because the level of credit is nonstationary in the data. Alternatively, one could use credit/GDP and results would be similar.

[20] Using quarterly data in the period 1971:Q3 to 2018:Q4 yield similar results than the long samples considered in Table 2.

latter is the expected value of a given payoff at a certain point in the future.[21] I show the key economic forces driving the yield curve, elaborated above, are also consistent with the evidence about the term structure of conditional distributions of future outcomes.

To compute the term-structure of distributions, consider a process z_t in the model that follows an Ito process

$$\mathrm{d}z_t = \mu_{z,t}\mathrm{d}t + \sigma_{z,t}\mathrm{d}W_t,$$

where $\mu_{z,t} = \mu_z(x_t)$ and $\sigma_{z,t} = \sigma_z(x_t)$ are the drift and diffusion. Next, I define the function $f(x_s|x_t = x^*, s)$ as the conditional distribution of x at each point in time $s > t$, starting from a point x^*. The evolution of the density over time can be described by the following partial differential equation

$$\frac{\partial f(z(x)|x^*, t)}{\partial t} = -\frac{\partial}{\partial x}\left[f(z(x)|x^*, t)\,\mu(x)\right] + \frac{1}{2}\frac{\partial^2}{\partial x^2}\left[f(z(x)|x^*)\,\sigma(x)^2\right],$$

which is also known as the forward Kolmogorov equation (or Fokker-Planck equation).

Figure 2 shows the forecasted conditional distributions for consumption (top two panels) and the state variable x (bottom two panels) at different horizons. The blue line represents the forecasted densities conditional on current financial conditions being loose. More precisely, the distributions are forecasted conditional on $x_t = x^*$, where x^* is 10% above the point at which financing constraints bind. The distribution in red represents the forecasted density conditional on current financial conditions being tight. I assume x_t is 10% below the point where financing constraints bind. The red dashed line in all four panels represents the point in the state space at which financing constraints

[21] In technical terms, these two objects are the forward Kolmogorov equation and the backward Kolmogorov equations.

bind.

In line with the evidence reported in Adrian et al. (2019), the top two panels indicate that, conditional on the economy facing tight financial conditions, the distribution of future consumption is negatively skewed. Also, the conditional distribution fluctuates across the horizon, and the relatively negative skewness of the conditionally constrained distribution persists even in the 3-year ahead forecasted distribution. The main source of the asymmetry between the constrained and unconstrained distributions is that economic outcomes are quite different in the constrained and unconstrained regions. For example, in the constrained region the economy is more leveraged (thus more sensitive to shocks), the real rate is much more volatile, and the price of risk moves faster. These conditions may persist because it takes time for financiers' wealth to be rebuilt. Simply put, x is a persistent process.

The bottom two panels display the forecasted conditional distributions for the state variable x. The intuition is similar to that of consumption growth. Tight financial conditions are persistent and can trigger quite volatile and unstable outcomes. Simply put, the model rationalizes the data with two main elements: Tighter financial conditions are persistent outcomes, and they lead to quite different economic outcomes than those implied by the economy functioning in an unconstrained region. As with the yield curve, the key elements are the bimodal nature of the economy and the persistent dynamics of intermediaries' wealth.

4 Conclusion

Financial intermediaries hold long-term assets, which means fluctuations in the long-term yields affect the extent to which intermediaries are financially constrained. These constraints affect marginal valuations in the economy, not only of the intermediaries, but in general equilibrium they affect other agents' marginal valuations as well. Hence, because long-term yields are forecasts of marginal valuations, financing constraints and long-term yields are directly related with each other.

In this paper, I show that financing constraints generate an endogenously time-varying real term premium that is consistent with the data. The nominal yield curve is primarily driven by the real factor that captures the health on intermediaries' balance sheets. The mechanism I propose can rationalize relevant yield curve facts, such as an upward-sloping real yield curve and highly volatile long-term yields, which are indeed hard to capture in standard macro models (Duffee, 2018). These results are purely driven by the fact that intermediaries' financing constraints may occasionally bind, linking the yield curve with intermediaries' financial health.

The novel economic mechanism I propose, connecting intermediaries' wealth and the yield curve, can rationalize interesting macroeconomic phenomena, suggesting that there are several potential avenues for further research. In particular, I show that a changes in the slope of the yield curve affects the business cycle because the slope of the yield curve is connected to intermediaries' willingness to engage in maturity transformation (and hence credit supply). Additionally, I show that the same mechanism explaining the term structure of interest rates is also able to rationalize the negative skewness in the term structure of distributions of consumption when financing constraints are binding.

However, there are several important quantitative elements of yields that the model cannot rationalize. For example, the model implies a positive correlation between bond and stock returns, while the evidence show the sign of such correlation has changed from positive to negative in the last few decades (Chernov, Lochstoer and Song (2023); Campbell, Pflueger and Viceira (2020)). Also, the model cannot rationalize the time-variation in yields' conditional skewness, an important property of yields that helps in understanding the time-variation of bond returns (Bauer and Chernov (2023)). Indeed, the model implies that conditional skewness is always positive and such conditional skewness has very little power for predicting bond returns. I speculate that incorporating additional state variables—for example by incorporating heterogenous intermediaries and savers—could help in addressing some of these issues.

5 Tables and Figures

TABLE 1. Calibration

PARAMETERS

		Value	Description
1. Preferences and Endowment			
	γ	5	Risk aversion
	ψ	1/5	EIS
	μ	0.007	Drift Y_t
	σ	0.036	Volatility Y_t
2. Financiers			
	λ	0.08	Dividend payout
	ω	0.85	Management cost
	κ	0.4	Fraction divertible assets
3. Nominal rate			
	δ_π	1.5	Taylor coefficient
	λ_s	0.06	Persistence of monetary shocks
	σ_s	0.0024	Volatility of monetary shocks

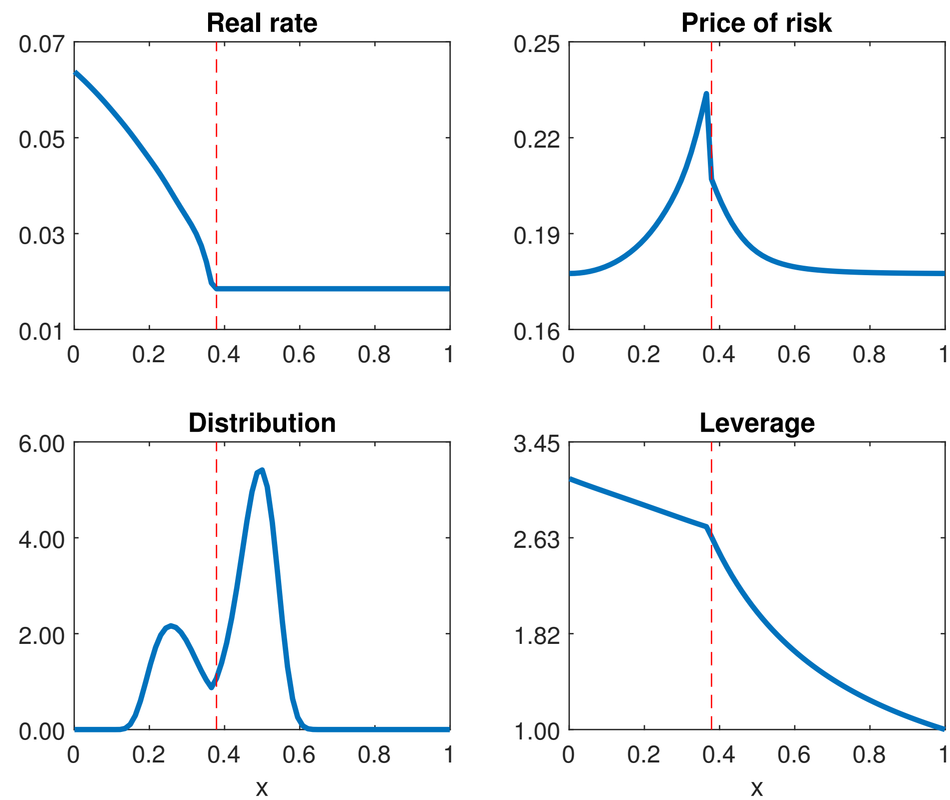
NOTES: This table shows the calibration of the model at an annual frequency.

TABLE 2. Slope of the yield curve and credit

	Dependent variable: $\Delta credit_t$					
	Full sample (1870-2016)			Postwar (1946-2016)		
Regressors	(1)	(2)	(3)	(4)	(5)	(6)
$\Delta slope_{t-1}$	0.922**	0.909**	0.745**	1.182**	1.172**	0.831*
	(0.386)	(0.421)	(0.366)	(0.575)	(0.613)	(0.450)
$\Delta credit_{t-1}$	0.598***	0.600***	0.609***	0.517***	0.523***	0.510***
	(0.091)	(0.081)	(0.083)	(0.176)	(0.155)	(0.146)
Δgdp_{t-1}		-0.014	-0.027		-0.034	-0.070
		(0.897)	(0.110)		(0.228)	(0.200)
$slope_{t-1}$			0.323*			0.607**
			(0.187)			(0.278)
R^2	0.344	0.344	0.349	0.311	0.316	0.345
Obs.	135	135	135	70	70	70

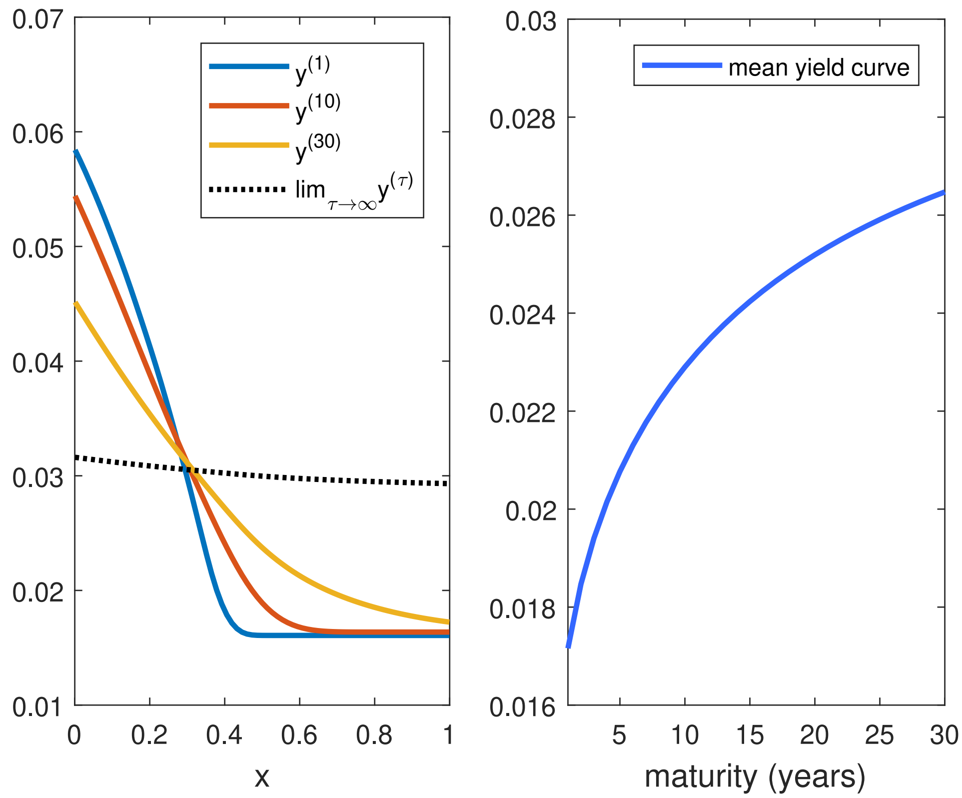
NOTES: The source of the data is Jordà et al. (2016). The dependent variable is $\Delta credit_t$, the log difference of real credit (Total loans to non-financial private sector over CPI) from $t-1$ to t. All regressions from (1) to (6) include a constant (not reported). Heteroskedasticity- and autocorrelation consistent asymptotic standard errors reported in parentheses are computed according to Newey and West (1987) with the automatic lag selection method of Newey and West (1994): *p<0.10; **p<0.05; and ***p<0.01.

FIGURE 1. Model Solution



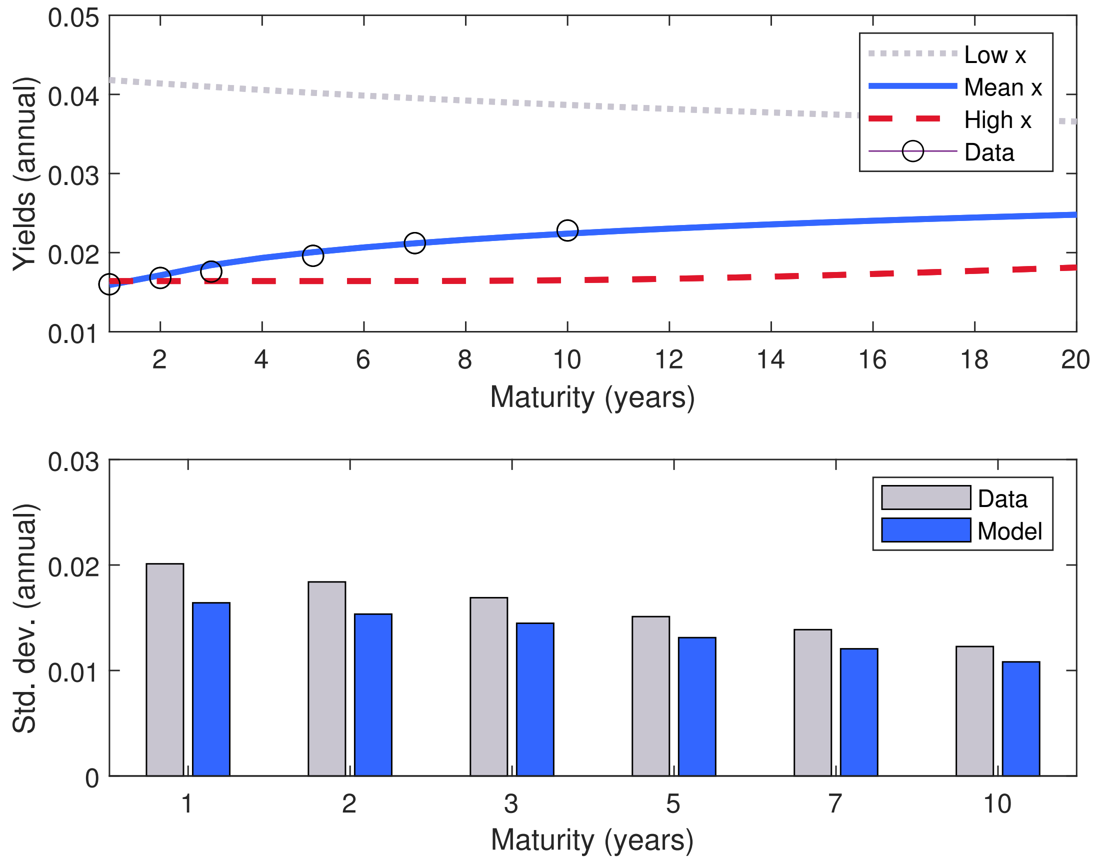
NOTES: This figure shows the model solution, with the calibrated parameters from Table 1. The red-dashed line represents the point of the state space in which the financial constraint binds.

FIGURE 2. Model Solution: Yields



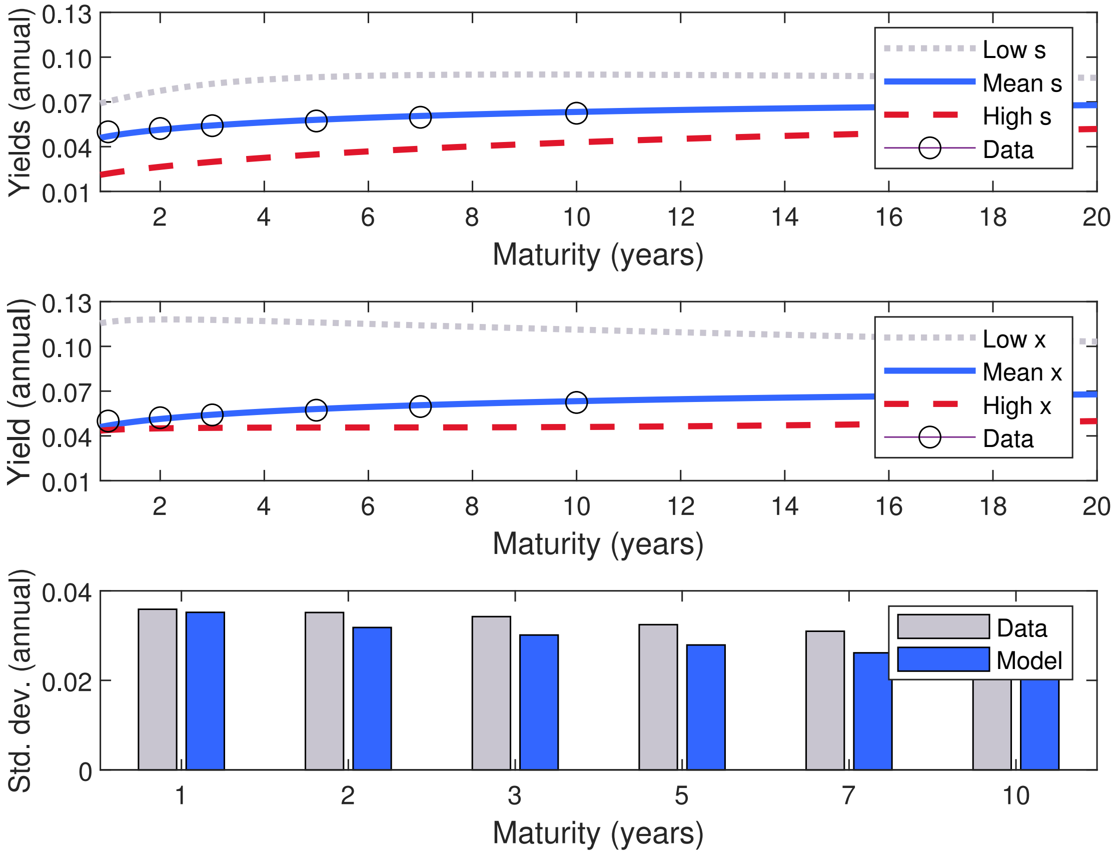
NOTES: This figure shows the real yield curve in the model, with the calibrated parameters from Table 1. The left panel shows yields for different maturities over the state-space. The right panel shows the average yields.

FIGURE 3. Real Yield Curve



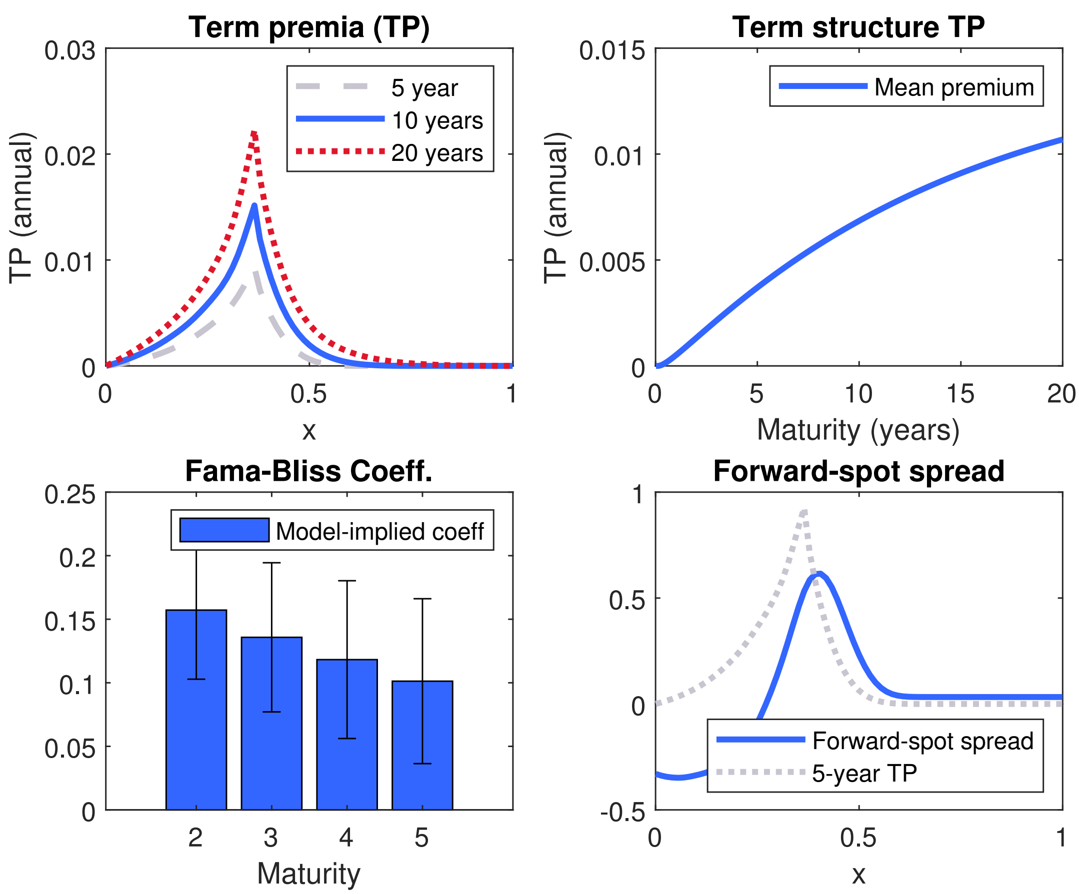
NOTES: This figure shows the real yield curve in the model, with the calibrated parameters from Table 1. The top panel displays the yield curve for three different levels of x (mean, and +- 2 standard deviations). The bottom panel show the standard deviation of yields. The data for real yields from Chernov and Mueller (2012) for the 1971-2002 period and TIPS data from Gürkaynak, Sack and Wright (2010) for the 2003-2018. See the appendix for more details about the data.

FIGURE 4. Nominal Yield Curve



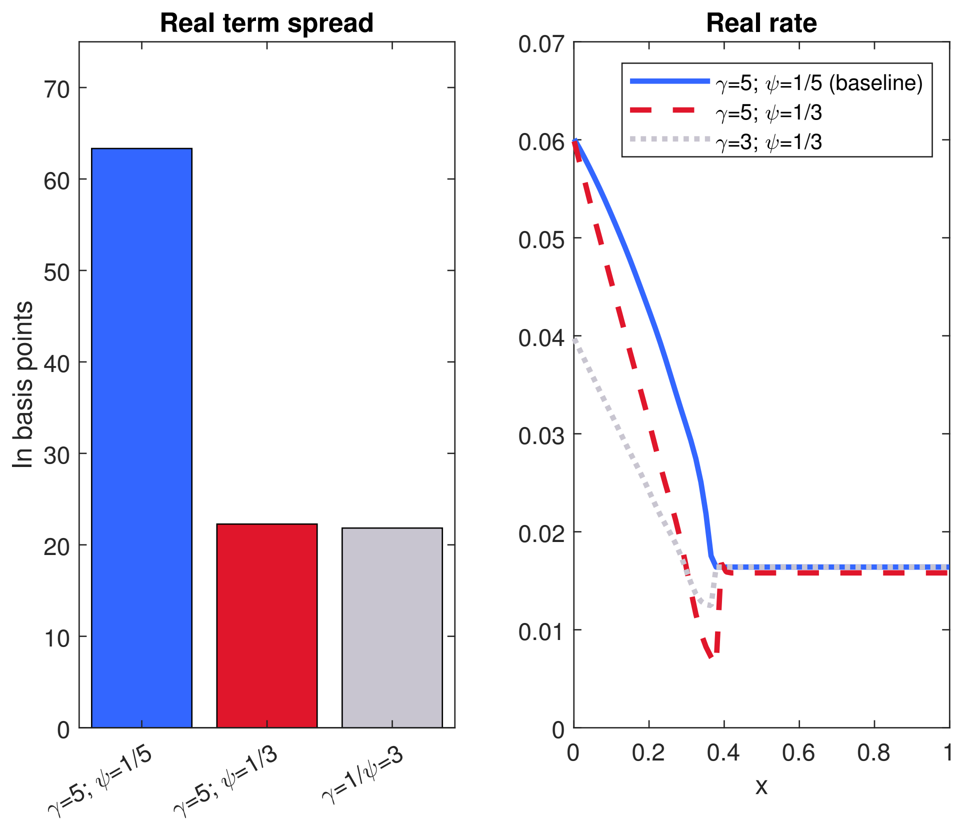
NOTES: This figure shows the nominal yield curve in the model, with the calibrated parameters from Table 1. The top panel displays the yield curve for three different levels of the persistent monetary policy shock variable s (mean, and +- 2 standard deviations), when x is at its steady state value. The bottom panel displays the yield curve for three different levels of the persistent state variable x (mean, and +- 2 standard deviations), when s is at its steady state value. The data for nominal yields is from Gürkaynak, Sack and Wright (2007). I provide details about the data in the appendix.

FIGURE 5. Term Premium and Bond Return Predictability



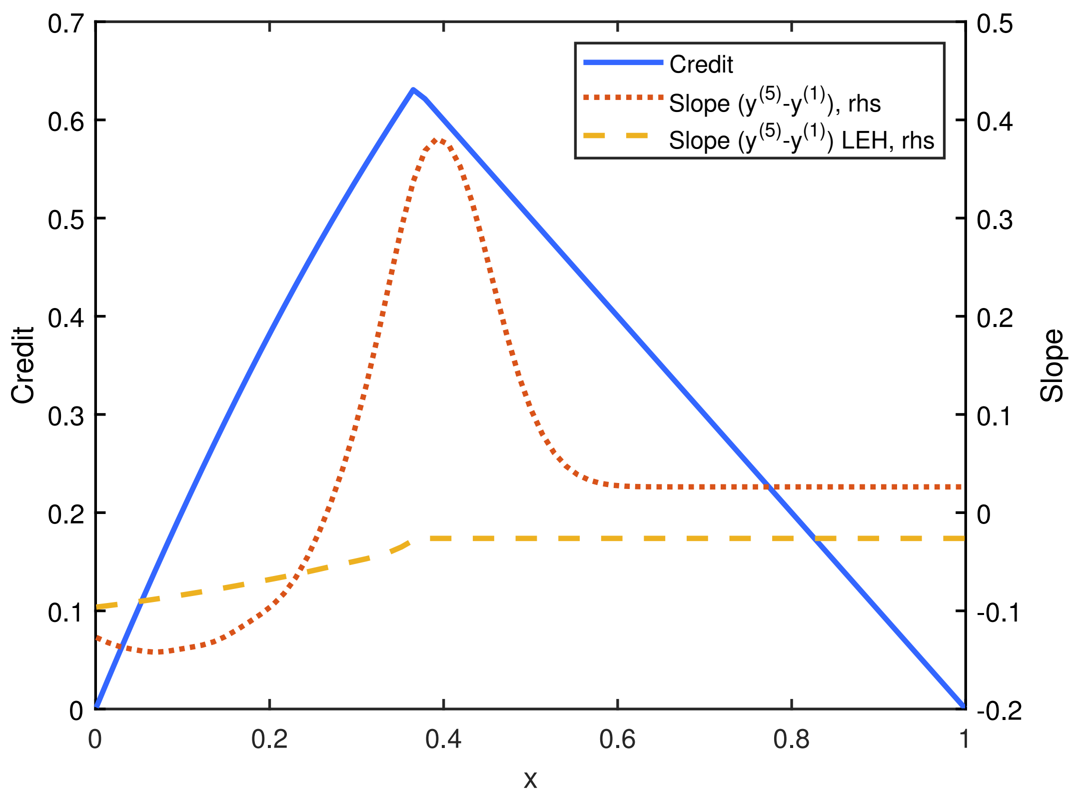
NOTES: The upper-left panel shows the term premium of zero-coupon bonds of different maturity. The upper-right panel shows the term structure of term premium when the state variable x is at its unconditional mean. The bottom-left panel shows the estimated coefficients of the Fama-Bliss regressions (shown in the main text) for simulated data from the model. The simulation consists of 200 paths with 100,000 realizations in each path. The confidence intervals are the 5th and 95th percentile of the estimated coefficients in each path. The lower-right panel shows the 5-year term premium and the forward-spot spread in the model.

FIGURE 6. The Role of Risk Aversion and the EIS



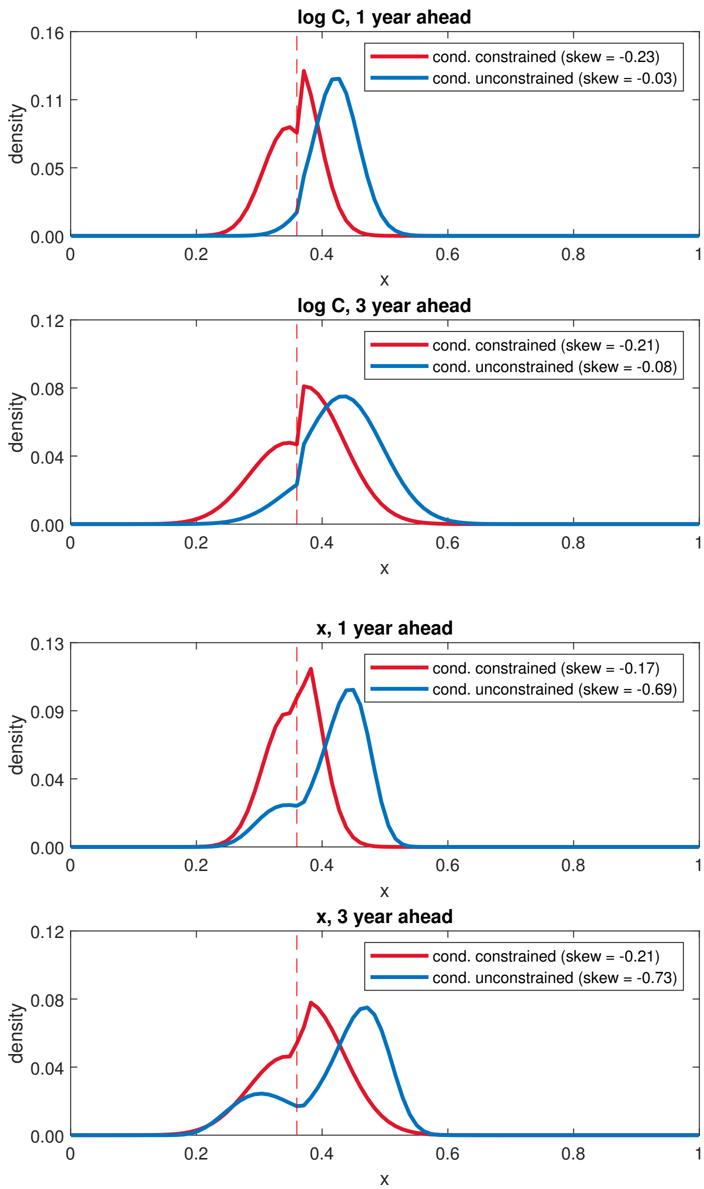
NOTES: The left panel shows the slope of the real yield curve (10-year minus 1-year spread) for different parametrizations of the risk aversion and EIS coefficients. The right panel shows the real rate across the state space for different parametrizations of the risk aversion and EIS coefficients, rescaled such that the real rate is the same in the first best for all parametrizations. The baseline calibration is the one reported in Table 1, namely $\gamma = 5$ and $\psi = 1/5$.

FIGURE 7. Slope and Credit



NOTES: This figure shows the slope of the yield curve and the credit in the model. Credit is the position of financiers' short-term deposits as a share of total net worth.

FIGURE 8. Term Structure of Conditional Distributions for Consumption



NOTES: This figure shows the conditional distributions of consumption 1 and 3 years ahead (top two panels), and the conditional distributions of the endogenous state variable x 1 and 3 years ahead (bottom two panels). The distributions are conditional on the state of the economy being an unconstrained one (blue lines) or a constrained one (red lines). The red dashed line is the point in the state space at which constraints binds.

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7 Appendix

Proof of Proposition 2. I derive the law of motion of the endogenous state variable, x_t, which is key in solving the ODEs. Applying Itô's lemma to $\frac{n_{f,t}}{q_t}$:

$$\mathrm{d}x_t = \frac{\mathrm{d}n_{f,t}}{q_t} - \frac{n_{f,t}}{q_t^2}\mathrm{d}q_t + \frac{n_{f,t}}{q_t}(\mathrm{d}q_t)^2 - \frac{\mathrm{d}q_t}{q_t}\mathrm{d}n_{f,t} + \lambda(\bar{x}q_t - \lambda n_{f,t})\mathrm{d}t, \tag{16}$$

where the last term, $\lambda\bar{x}q_t - \lambda n_{f,t}$, represents the initial capital received by intermediaries, net of their aggregate dividends. Then, substituting the law of motion for q_t and $n_{f,t}$ in (16), I get

$$\begin{aligned}\mathrm{d}x_t &= x_t\left[r_t + \frac{q_t\theta_{f,t}}{n_{f,t}}\left(\mathbb{E}_t\left[\mathrm{d}R_{f,t}\right] - r_t\right) - \mu_{q,t} + \left(\frac{q_t\theta_{f,t}}{n_{f,t}} - 1\right)\sigma_{q,t}^2\right]\mathrm{d}t + \\ &\quad x_t\left(\frac{q_t\theta_{f,t}}{n_{f,t}} - 1\right)\sigma_{q,t}\mathrm{d}W_t + \lambda\left(\bar{x} - x_t\right)\mathrm{d}t.\end{aligned}$$

Based on the previous expression, I denote the drift and diffusion as

$$\begin{aligned}x_t\mu_{x,t} &= x_t\left[r_t + \frac{q_t\theta_{f,t}}{n_{f,t}}\left(\mathbb{E}_t\left[dR_{f,t}\right] - r_t\right) - \mu_{q,t} + \left(\frac{q_t\theta_{f,t}}{n_{f,t}} - 1\right)\sigma_{q,t}^2\right] + \lambda\left(\bar{x} - x_t\right), \\ x_t\sigma_{x,t} &= x_t\left(\frac{q_t\theta_{f,t}}{n_{f,t}} - 1\right)\sigma_{q,t}.\end{aligned}$$

I next derive the system of ordinary differential equations for 5 unknown functions, namely the price-dividend ratio $p(x)$, the financier's marginal value of $\phi(x)$, the saver's utility function $\xi(x)$, the real bond prices $\{P(x,\tau)\}_{\tau\geq 0}$, and the nominal bond prices $\{P^{\$}(x,s,\tau)\}_{\tau\geq 0}$.

When the financing constraint is not binding, the first order conditions of financier's problem is, when $\theta_{f,t} > 0$,

$$\mathbb{E}_t\left[\mathrm{d}R_{f,t}\right] - r_t \mathrm{d}t + \mathbb{E}_t\left[\left(\frac{\mathrm{d}m_t}{m_t} + \frac{\mathrm{d}\phi_t}{\phi_t}\right)\mathrm{d}R_{f,t}\right] = 0. \tag{17}$$

Equation (17) is the first equation in proposition 2 (i.e., the ODE for the price-dividend in the unconstrained region). Then substituting (17) into the financier's value function (7) yields the second equation in the system (i.e., the ODE for financier's marginal value of wealth, ϕ). When financiers is constrained, the savers' pricing equation is

$$\mathbb{E}_t\left[\mathrm{d}R_{s,t}\right] - r_t \mathrm{d}t + \mathbb{E}_t\left[\frac{\mathrm{d}m_t}{m_t}\mathrm{d}R_{s,t}\right] = 0. \tag{18}$$

Equation (18) is the fourth equation in proposition 2, and replacing (18) into the financiers' value function gives the fifth equation.

Finally, the third and sixth equations are the savers' value function. I denote the value function with optimal policies as (c^*, U^*)

$$0 = \frac{1}{1-\frac{1}{\psi}}\left\{\rho\left(c^*\right)^{1-\frac{1}{\psi}}\left[(1-\gamma)U^*\right]^{\frac{\frac{1}{\psi}-\gamma}{1-\gamma}} - \rho(1-\gamma)U^*\right\} + E_t\left[\mathrm{d}U^*\right].$$

I guess and verify the solution is given by

$$U^* = \frac{\left(\xi(x)c^*\right)^{1-\gamma}}{1-\gamma}.$$

Then, using Itô's lemma and substituting, I get

$$0 = \frac{\rho}{1-\frac{1}{\psi}}\left\{\xi^{\frac{1}{\psi}-1} - 1\right\} + \mu_c - \frac{1}{2}\sigma_c^2 + \frac{\xi_x}{\xi}x\mu_x - \frac{\gamma}{2}\left(\frac{\xi_{xx}}{\xi}x\sigma_x\right)^2 + (1-\gamma)\frac{\xi_x}{\xi}x\sigma_c.$$

All drifts and diffusions terms in proposition 2, terms $\mu_p(x), \mu_\phi(x), \mu_\xi(x), \mu_P(x,\tau), \mu_c(x)$ and $\sigma_p(x), \sigma_\phi(x), \sigma_\xi(x), \sigma_P(x,\tau), \sigma_c(x)$ are partial derivatives from applying Itô's lemma in their corresponding functions. That is,

$$\begin{aligned}
\mu_p(x) &= \frac{p_x}{p}\mathbb{E}_t[\mathrm{d}x] + \frac{1}{2}\frac{p_{xx}}{p}\mathbb{E}_t\left[\mathrm{d}x^2\right], \\
\mu_\phi(x) &= \frac{\phi_x}{\phi}\mathbb{E}_t[\mathrm{d}x] + \frac{1}{2}\frac{\phi_{xx}}{\phi}\mathbb{E}_t\left[\mathrm{d}x^2\right], \\
\mu_\xi(x) &= \frac{\xi_x}{\xi}\mathbb{E}_t[\mathrm{d}x] + \frac{1}{2}\frac{\xi_{xx}}{\xi}\mathbb{E}_t\left[\mathrm{d}x^2\right], \\
\mu_P(x,\tau) &= \frac{P(\tau)_x}{P(\tau)}\mathbb{E}_t[\mathrm{d}x] + \frac{1}{2}\frac{P(\tau)_{xx}}{P(\tau)}\mathbb{E}_t\left[\mathrm{d}x^2\right],
\end{aligned}$$

and

$$\sigma_p(x) = \frac{p_x}{p}x\sigma_x; \sigma_\xi(x) = \frac{\xi_x}{\xi}x\sigma_x; \sigma_\xi(x) = \frac{\xi_x}{\xi}x\sigma_x; \sigma_P(x,\tau) = \frac{P(\tau)_x}{P(\tau)}x\sigma_x.$$

The drift and difussion for the nominal bond prices can be expressed in a similar way, in terms of the partial derivatives, but include additional the extra terms associated with of the exogenous state variable—the monetary policy shocks.

For consumption, use the market clearing condition for goods

$$\begin{aligned}
c_t &= \left[\omega\alpha_t^s(1-x_t) + x_t\alpha_t^f\right]y_t, \\
&= \left[\omega + (1-\omega)x_t\alpha_t^f\right]y_t,
\end{aligned}$$

and apply Itô's lemma:

$$\mu_c = \frac{(1-\omega)\, x_t \alpha_t^f}{\left[\omega + \ (1-\omega)\, x_t \alpha_t^f\right]} \left[\mu_x + \frac{\phi_x}{\phi} x \mu_x + \frac{1}{2}\frac{\phi_{xx}}{\phi}(x\sigma_x)^2 + \frac{\phi_x}{\phi} x \sigma_x^2 + \left(\frac{\phi_x}{\phi} x + 1\right)\sigma_x \sigma\right] + \mu,$$

$$\sigma_c = \frac{(1-\omega)\, x_t \alpha_t^f}{\left[\omega + \ (1-\omega)\, x_t \alpha_t^f\right]} \left(\frac{\psi_x}{\psi} x + 1\right)\sigma_x + \sigma.$$

Finally, I show $r_t \mathrm{d}t = -\mathbb{E}_t\left[\frac{\mathrm{d}m_t}{m_t}\right]$. For this, I follow the characterization in Cox, Ingersoll and Ross (1985a) and Duffie and Epstein (1992a). Savers' problem is

$$0 = \max_{c,\theta_s} \frac{1}{1-\frac{1}{\psi}} \left\{\rho c^{1-\frac{1}{\psi}} \left[(1-\gamma)\, U\right]^{\frac{\frac{1}{\psi}-\gamma}{1-\gamma}} - \rho\,(1-\gamma)\, U\right\} + \mathbb{E}_t\left[\mathrm{d}U\right]. \tag{19}$$

subject to (2). Because $U(x,n)$, I write

$$\mathbb{E}_t\left[\mathrm{d}U\right] = U_x \mathbb{E}_t\left[\mathrm{d}x\right] + \frac{1}{2} U_{xx} \mathbb{E}_t\left[\mathrm{d}x^2\right] + U_n \mathbb{E}_t\left[\mathrm{d}n\right] + \frac{1}{2} U_{nn} \mathbb{E}_t\left[\mathrm{d}n^2\right] + U_{nx} \mathbb{E}_t\left[\mathrm{d}n \mathrm{d}x\right].$$

The first order conditions (FOC) are

$$\rho c^{-\frac{1}{\psi}} \left[(1-\gamma)\, U\right]^{\frac{\frac{1}{\psi}-\gamma}{1-\gamma}} = U_n,$$

$$q U_n \left(\mathbb{E}_t\left[\mathrm{d}R^s\right] - r\right) + U_{nn} \theta_s q^2 \sigma_q^2 + U_{xn} q \sigma_q \sigma_x = 0.$$

Then, I substitute the law of motion for n, and the first order conditions in (19) to get

$$\begin{aligned} 0 = {} & \frac{\rho^\psi}{1-\frac{1}{\psi}} \left[(1-\gamma)\, U\right]^{\frac{\frac{1}{\psi}-\gamma}{1-\gamma}\psi} U_n^{1-\psi} - \frac{\rho\,(1-\gamma)\, U}{1-\frac{1}{\psi}} - \rho^\psi \left[(1-\gamma)\, U\right]^{\frac{\frac{1}{\psi}-\gamma}{1-\gamma}\psi} U_n^{1-\psi} \\ & + U_n\left[T + rn\right] - \frac{1}{2} U_{nn} \left(\theta^s q\right)^2 \sigma_q^2 + U_x \mathbb{E}_t\left[\mathrm{d}x\right] + \frac{1}{2} U_{xx} \mathbb{E}_t\left[\mathrm{d}x^2\right]. \end{aligned} \tag{20}$$

The next step is to take the derivative of (20) with respect to n. After some algebra,

$$\begin{aligned}
0 \quad = \quad & \left[\left(\frac{\frac{1}{\psi}-\gamma}{1-\gamma}\right)\left(\frac{\psi}{\psi-1}\right)\frac{U_n}{U}-\frac{U_{nn}}{U_n}\right]\rho^{\psi}\left[(1-\gamma)U\right]^{\frac{\frac{1}{\psi}-\gamma}{1-\gamma}\psi}U_n^{1-\psi} \\
& -\frac{\rho(1-\gamma)}{1-\frac{1}{\psi}}U_n \\
& +U_{nn}\left[T+rn\right]+U_n r \\
& -\frac{1}{2}U_{nnn}\left(\theta^s q\right)^2\sigma_q^2+U_{nx}\mathbb{E}_t\left[\mathrm{d}x\right]+\frac{1}{2}U_{nxx}\mathbb{E}_t\left[\mathrm{d}x^2\right]. \qquad (21)
\end{aligned}$$

The final step is to subtract the stochastic discount factor from the expression above. For this, following Duffie and Epstein (1992a),

$$\frac{\mathrm{d}m_t}{m_t}=\frac{\mathrm{d}f_c}{f_c}+f_U\,\mathrm{d}t,$$

with

$$\begin{aligned}
f_U \quad &= \quad \frac{\frac{1}{\psi}-\gamma}{1-\frac{1}{\psi}}\rho^{\psi}\left[(1-\gamma)U\right]^{\frac{(1-\psi)\gamma}{1-\gamma}}U_n^{1-\psi}-\frac{\rho(1-\gamma)}{1-\frac{1}{\psi}}, \\
f_c \quad &= \quad \rho c^{-\frac{1}{\psi}}\left[(1-\gamma)U\right]^{\frac{\frac{1}{\psi}-\gamma}{1-\gamma}}.
\end{aligned}$$

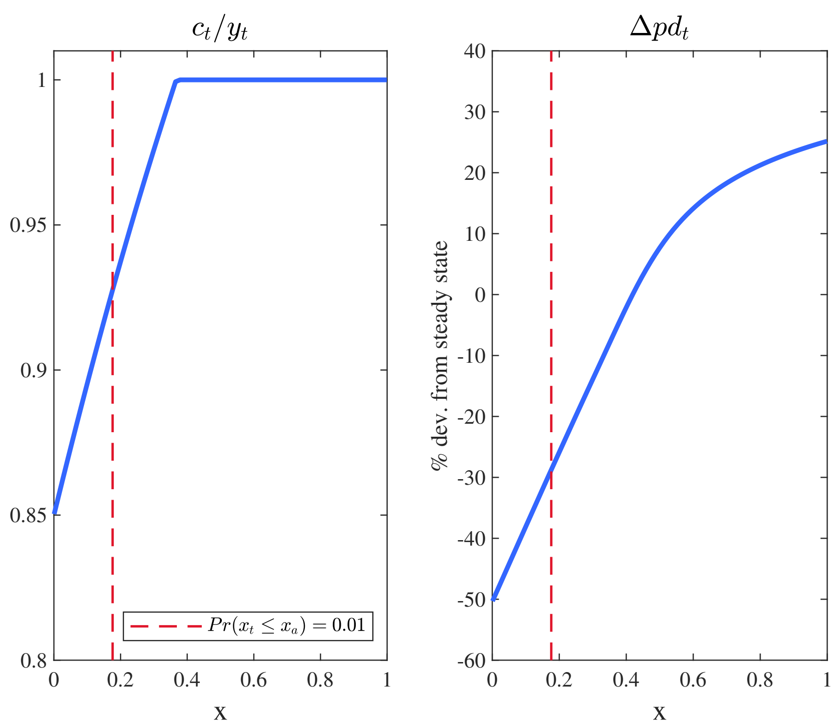
From FOC, I have that $f_c=U_n$, so the SDF m_t can be written as

$$\begin{aligned}
\frac{\mathrm{d}m}{m} \quad = \quad & \frac{U_{nn}}{U_n}\left[rn+T\right]-\rho^{\psi}\left[(1-\gamma)U\right]^{\frac{\frac{1}{\psi}-\gamma}{1-\gamma}\psi}U_n^{-\psi}\frac{U_{nn}}{U_n}-\frac{1}{2}\frac{U_{nnn}}{U_n}\left(\theta q\right)^2\sigma_q^2 \\
& +\frac{U_{nx}}{U_n}\mathbb{E}_t\left[\mathrm{d}x\right]+\frac{1}{2}\frac{U_{nxx}}{U_n}\mathbb{E}_t\left[\mathrm{d}x^2\right] \\
& +\frac{\frac{1}{\psi}-\gamma}{1-\frac{1}{\psi}}\rho^{\psi}\left[(1-\gamma)U\right]^{\frac{(1-\psi)\gamma}{1-\gamma}}U_n^{1-\psi}-\frac{\rho(1-\gamma)}{1-\frac{1}{\psi}}. \qquad (22)
\end{aligned}$$

Finally, subtract (21) from (22) and obtain $\mathbb{E}_t\left[\frac{\mathrm{d}m}{m}\right] = -r_t \mathrm{d}t$.

Consumption and price-dividend during financial disruptions. The figure below shows the consumption losses (left panel) and changes in the price-dividend ratio (right panel) across the state space.[22] The vertical red-dashed line shows the point in the state space at which the invariant distribution accumulates is 1% probability.

FIGURE A.1. Consumption and Price-Dividend



As shown in the figure A.1, it is very unlikely that, in the model's equilibrium, consumption losses will exceed -7% and the price-dividend ratio changes exceed -28.8% in a given year. Muir (2017) shows that declines in consumption and the price-dividend ratio associated with financial crises—defined as a banking panic or banking crisis—are broadly in line with the model: the evidence for financial crises indicates that those

[22]In equilibrium, $\frac{c_t}{y_t} = \omega + (1-\omega)\, x_t \alpha_t^f$.

episodes are associated with an average decline of approximately 25% in price-dividend ratio and a 9% decline in consumption levels.[23] Additionally, Greenwood et al. (2022) show, using a very similar dataset than Muir (2017), that the unconditional probability of a financial crises is 4%.[24] Hence, the evidence points to rare occasions (4% probability) in which financial intermediaries wealth is unpaired and those occasions are characterized by approximately 9% consumption losses and a 25% decline in the price-dividend ratio, which similar to the model implications.

Data. The source of data for real yields used in Figure (2) is Chernov and Mueller (2012) during the period 1971-2002 and Gürkaynak et al. (2010) for TIPS during 2003-2018. The data from Chernov and Mueller (2012) can be easily accessed at the Muller's website. The data for nominal yields in Figure 4 is from Gürkaynak et al. (2007) for the period 1971-2018. The table below shows the mean and standard deviation of real and nominal yields, which are the statistics used in figures 2 and 4. The following table shows the summary statistics, expressed in decimals at an annual frequency.

TABLE A.1. Summary Statistics on TIPS and Nominal Treasuries.

		1y	2y	3y	5y	7y	10y
TIPS							
	Average	0.016	0.017	0.018	0.020	0.021	0.023
	St. Dev.	0.019	0.017	0.016	0.015	0.013	0.012
Nominal							
	Average	0.052	0.054	0.056	0.059	0.062	0.064
	St. Dev.	0.035	0.034	0.033	0.032	0.030	0.029

[23] More details about the definition of financial crises can be found in Muir (2017), Section 2.
[24] Jordà et al. (2016)

Numerical method. I use a spectral collocation method based on Chebyshev polynomials of the first kind to solve the numerically solve the model. Conceptually, the numerical solution consists of representing the unknown functions as Chebyshev polynomials on a grid and then substitute them into the ODEs characterizing the equilibrium. In particular, I solve for: i) The financiers' value functions; ii) the price-dividend ratio; iii) the savers' value function; iv) real bond prices; and v) nominal bond prices. The system of equations is in proposition 2 in the text and the detailed derivation is in the appendix.

The steps are as follow

1. First, construct a grid with K Chebyshev nodes

$$h_i = \cos\left(\frac{2i+1}{2(K+1)}\pi\right),\ i = 0, ..., K.$$

   Therefore, $h_i \in [-1, 1]$. Since in the model $x \in (0, 1)$, I express the grid on x as $x_i = \frac{1}{2}(1 + h_i)$.

2. Then, a given function $g(x)$ that needs to be solved, can be written in a polynomial form

$$g(x) = \sum_{i=0}^{K} a_i \Psi_i(h_i(x)) + O(K),$$

   where K is the order of the polynomial, Ψ is the basis function (which in this case is the Chebyshev polynomials), $\{a_i\}_{i=0}^{K}$ are unknown coefficients that need to solved, h_i are the Chebyshev nodes, and $O(K)$ is an approximation error.

3. Then, solve for the associated set of unknown coefficients $\{a_i\}_{i=0}^{K}$ in each function, such that equilibrium conditions are verified.

   (a) Start with a guess for the unknown coefficients.

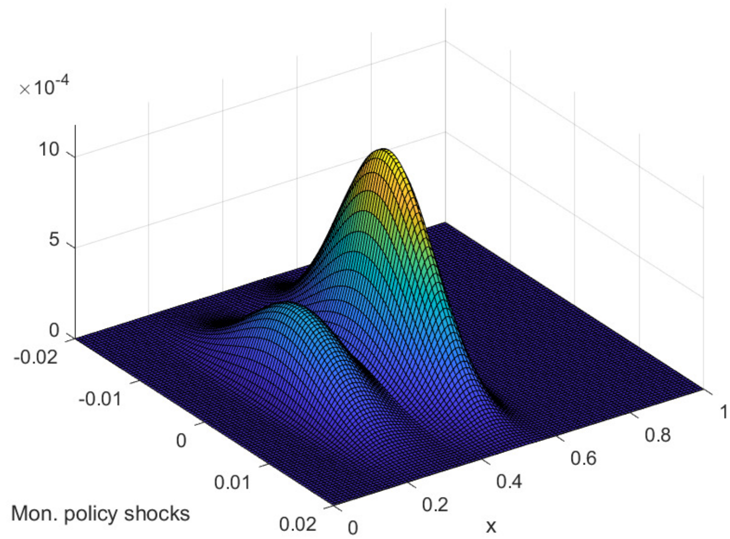
(b) For a guessed solution, use financiers' optimality condition to compute x^*, which is the point at which the leverage constraint binds. That is, the point at which $\alpha_{f,t} = 1/x_t$ becomes $\alpha_{f,t} = \phi_t/\kappa$.

(c) Solve the corresponding system of equations with a non-linear solver and verify.

4. Once the equilibrium is solved, I solve the yield curve using forward finite difference across the maturity dimension. That is, starting from $P^{(0)}(x) = 1$, solve for the bond prices in the state space (using the steps above) and iterate across the maturity dimension with

$$\frac{P^{(\tau+\Delta)}(x) - P^{(\tau)}(x)}{\Delta} \approx P_\tau^{(\tau)}(x).$$

FIGURE A.2. Bi-variate distribution of x and monetary policy shocks.



NOTES: This figure shows the invariant distribution of the endogenous state variable, x, and the exogenous state variable, monetary policy shocks.