Financial Intermediaries and the Yield Curve

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Abstract

I study the yield curve dynamics in a general equilibrium model with financial intermediaries facing financing constraints. When constraints bind, intermediaries reallocate their portfolio, causing deadweight losses in aggregate consumption, thus affecting savers' marginal utility. Because the yield curve is a forecast of marginal utility, intermediaries constraints show up, via general equilibrium forces, in long-term yields. I show that the mechanism connecting intermediaries constraints and long-term yields produces highly nonlinear interest rate dynamics and a positive real term premium in equilibrium. I extend the analysis to the nominal yield curve using a simple Taylor rule.

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One of the main lessons of the large body of research on the nexus between financial intermediation and the macroeconomy is that financial intermediaries face constraints that distort the allocation of goods and capital—hence affecting agents' marginal valuations. In this paper, I argue that the yield curve contains information about such distortions because long-term yields are, by definition, a forecast of the economy's marginal valuation (Alvarez and Jermann (2005)). In other words, intermediaries' constraints are a macro source of term premia, which means long-term yields and intermediaries' balance sheet dynamics are closely related.

More precisely, I study a canonical general equilibrium intermediary asset pricing model to underscore the mechanism through which intermediaries' constraints produce a positive term premium. I show that the connection between intermediaries' constraints, marginal utility, and long-term yields help us in rationalizing the salient properties of the U.S. real and nominal yield curves. In particular, the model features highly nonlinear real yields, with an average upward-sloping real and nominal yield curve and highly volatile long-term yields, facts that have proven very difficult to match in representative agent models (Duffee (2018)). These results are purely driven by the fact that financial intermediaries face occasionally binding constraints. Indeed, if intermediaries were always unconstrained, then the yield curve would be flat and constant.

The mechanism is grounded in two main elements: Intermediaries operate with leverage in equilibrium (i.e., there is borrowing and lending between savers and intermediaries), and they face financing constraints. These two elements have been extensively studied in the macro-finance literature, but in this paper, I focus the analysis on the yield curve.[1] To obtain leverage in equilibrium, I follow Brunnermeier and San-

[1] Recent literature, reviewed later, has departed from the representative agent analysis of the yield curve but without stressing the role of financing constraints—a salient characteristic of intermediaries.

nikov (2014), among others, and I assume intermediaries are more efficient than savers in handling risky assets. That is, financial intermediaries issue short-term deposits to savers to fund positions in long-term risky assets and take advantage of their relatively better investment technology. However, intermediaries' positions in long-term risky assets can be constrained in certain states of the world because of agency problems, as in Gertler and Kiyotaki (2015). As a consequence, when intermediaries hit their constraints, they are forced to sell risky assets to less efficient savers, and, subsequently, aggregate consumption and asset prices decline, the price of risk increases, and the wealth of financial intermediaries deteriorates even further, which force intermediaries to reallocate their portfolios, and so on. This well-known feedback mechanism has important implications for the yield curve, as I detail next.

The presence of occasionally binding constraints implies the economy features a bimodal distribution: It spends the vast majority of time in a "normal regime," in which constraints are slack, risk premia are low, the real interest rate is low, and volatility of asset prices is moderate. When negative aggregate shocks occur, the economy can enter a "crisis regime," in which financing constraints are binding. Here, intermediaries reallocate their portfolios, and wealth is transferred to inefficient savers. Savers' inefficiencies in handling risky assets cause deadweight losses and push the consumption level persistently below the trend growth, and, therefore, the real interest rate persistently increases as agents perceive the crisis regime as transitory—the consumption level will recover its trend in the future.[2] But this dynamic occurs precisely when the price of risk spikes, implying that real bond prices go down in states in which the marginal investor values those resources the most—a crisis regime. Thus, real bonds carry an endoge-

[2] I show that the nonlinear increase in the real rate when financing constraints binds depends mostly on the magnitude of savers' elasticity of intertemporal substitution.

nously time-varying term premium, and the yield curve is upward sloping, on average, due to the fact that there is always a nonzero probability the economy can hit financing constraints.

I extend the analysis to study the nominal yield curve by introducing a simple monetary policy rule that is subject to persistent monetary shocks. The monetary policy rule, which takes the form of a Taylor rule, pins down an equilibrium inflation process that depends on the state of the economy as well as on persistent monetary policy shocks. The equilibrium nominal yield curve is consistent with the empirical evidence as long as the reaction function of the monetary policy rule with respect to inflation is greater than one-for-one. As noted in Schneider (2022), if the model can capture the main properties of the real yield curve, then the nominal yield curve can be simply rationalized with an empirically plausible Taylor rule (i.e., a rule in which the monetary authority adjusts the interest rate more than one-for-one with inflation). I extend the analysis in Schneider (2022) to include persistent monetary policy shocks and therefore have a flexible environment in which the nominal yield curve is driven by both real and nominal shocks.

Besides accounting for the salient properties of the real and nominal yield curves (violation of the expectation hypothesis, a positive average term premium, highly volatile long-term yields, and the bond return predictability), I show that the mechanism relating intermediaries' wealth and the yield curve can be extended to study the term structure of the conditional distributions of macroeconomic outcomes. The purpose of this exercise is to illustrate that the mechanism in the model, grounded on occasionally binding constraints, can rationalize evidence beyond the scope of the yield curve, therefore providing external validation of the key economic forces in the model.

Recent literature has stressed the role of financial conditions in driving the distribution of real variables in the near future (Giglio, Kelly and Pruitt (2016); Adrian, Boyarchenko and Giannone (2019)). More precisely, when financial conditions deteriorate, the forecasted conditional distribution of GDP growth becomes more negatively skewed. Moreover, this distribution changes with the forecast horizon; there is a term structure of conditional distributions. The conditional distributions of real variables are intimately related to the yield curve, because long-term yields are a conditional expectation of future variables, while the forecasted distribution consists of computing the entire distribution of future realizations. To rationalize the evidence, I compute the term structure of the conditional probability density function of consumption and intermediaries' wealth. This is the model's theoretical counterpart of the estimated conditional distributions in, for example, Adrian et al. (2019). I show the model captures the evidence relatively well: Conditional on a state in which intermediaries are constrained (tight financial conditions), the term structure of conditional distributions of consumption exhibits a negative skewness. When financial conditions are loose, the negative skewness vanishes, and the term structure of conditional distributions is roughly Gaussian.

Related Literature. This paper relates to a strand of literature that has departed from the representative agent analysis of the yield curve. In this line, part of the literature has stressed the role of certain agents (arbitrageurs, intermediaries, etc.) in explaining the yield curve dynamics, typically in a partial equilibrium setup (Greenwood and Vayanos (2014); Haddad and Sraer (2019); Vayanos and Vila (2021)). Relative to this literature, the contribution of this paper is to use a general equilibrium framework to study the role of financing constraints in driving the yield curve dynamics.[3]

The general equilibrium framework I build on (see, for example, He and Krishnamurthy (2013), Brunnermeier and Sannikov (2014), and Gertler and Kiyotaki (2015), among many others) has been extensively studied in the macro-finance literature to answer a variety of questions, particularly after the Great Recession. For example, Gertler and Karadi (2011) study unconventional monetary policies, Van der Ghote (2021) studies the coordination of conventional and macroprudential policies, Maggiori (2017) studies the risk sharing dynamics between countries that differ in their degree of financial development, and Bigio and Schneider (2017) analyze the role of financing constraints and liquidity shocks in driving the equity premium. Relative to this literature, the contribution of this paper is to shift the focus away from equities (or "capital") to the yield curve dynamics. In particular, I show that financing constraints play a crucial role in producing an endogenously time-varying real term premium. Also, I show that the connection between the yield curve and financial intermediaries' wealth is important for understanding why tight financing constraints imply a negatively skewed distribution of future economic outcomes.

1 Model

I propose a general equilibrium model with a financial intermediary sector along the lines of He and Krishnamurthy (2013), Brunnermeier and Sannikov (2014), and Gertler and Kiyotaki (2015) and focus on the pricing implications for the yield curve. I first

[3]Other papers have studied the yield curve in a general equilibrium setup with heterogeneous agents (see for example, Wang (1996), Ehling, Gallmeyer, Heyerdahl-Larsen and Illeditsch (2018), and Schneider (2022), among others) but without financing constraints.

solve for the real equilibrium and derive the real yield curve. I next extend the analysis to include a monetary policy rule and derive the nominal yield curve.

Time is continuous and denoted by $t > 0$. Aggregate output, denoted by Y_t, follows

$$\frac{\mathrm{d}Y_t}{Y_t} = \mu \mathrm{d}t + \sigma \mathrm{d}W_t,$$

where parameters $\mu > 0, \sigma > 0$ are constants and W_t is a standard Brownian motion in a complete probability space (Ω, F, P) .

The economy is populated by a continuum of savers (denoted by s) and a continuum of financiers (denoted by f). Financiers are in charge of managing financial firms, which are owned by savers, while the savers maximize their discounted utility from consumption. Both f and s are allowed to trade risky assets, but a key difference between f and s is that the former have a comparative advantage over the latter in operating risky assets over the latter—which implies f and s engage in borrowing and lending in equilibrium.[4]

Agents can trade two classes of assets—namely, long-term risky assets and short-term risk-free deposits. The long-term asset is in exogenous fixed supply, and I denote its ex-dividend price by q_t. This asset pays a dividend Y_t per unit of time if held by f, but it pays $\omega Y_t, \omega < 1$, if held by s. That is, it is more costly for savers than financiers to operate this risky asset.[5] The purpose of this assumption is to obtain endogenous leverage in equilibrium, in the same way as in Brunnermeier and Sannikov (2014): Financiers

[4]Although my assumption about f and s having different expertise in handling the risky assets differs from that of He and Krishnamurthy (2013)—who assume that markets are segmented and that only "specialists" can trade risky assets—the asset pricing implications of both assumptions are similar (see Brunnermeier, Eisenbach and Sannikov (2013)).

[5]This assumption about ω is equivalent to assuming that savers have to pay a cost to operate risky assets (Gertler and Kiyotaki (2015)).

have an advantage in handling risky assets and therefore will borrow from savers to take leveraged positions in risky assets. A direct consequence of assuming $\omega < 1$ is that when financiers' wealth is impaired and savers are handling risky assets, the aggregate dividend (and, in equilibrium, consumption) will decline.

The total return of investing in the long-term asset consists of the dividend yield plus the capital gains. For financiers, this return is

$$\mathrm{d}R_{f,t} = \frac{Y_t}{q_t}\mathrm{d}t + \frac{\mathrm{d}q_t}{q_t},$$

while for savers, the total return is

$$\mathrm{d}R_{s,t} = \frac{\omega Y_t}{q_t}\mathrm{d}t + \frac{\mathrm{d}q_t}{q_t},\ \omega < 1.$$

Second, the short-term deposit account is in zero net supply, and it yields a risk-free interest rate, denoted by r_t. For simplicity, I solve the model with the generic long-term asset q_t and the short-term deposit account.Later, I introduce zero-coupon bonds of all maturities that are also in zero net supply. That is, zero coupons are redundant in the construction of the equilibrium, but they are useful to characterize the economy's equilibrium yield curve.

Savers consume and save. They have recursive preferences as in Duffie and Epstein (1992b) and their utility function is given by

$$U_t = \mathbb{E}_t\left[\int_t^{\infty} f\left(c_u, U_u\right)\mathrm{d}u\right],$$

where

$$f(c,U) = \frac{\rho}{1-\frac{1}{\psi}} \left\{ \frac{c^{1-1/\psi}}{\left[(1-\gamma)U\right]^{\left(\gamma-\frac{1}{\psi}\right)/(1-\gamma)}} - (1-\gamma)U \right\}. \tag{1}$$

In (1), c is savers' consumption, ρ is the time preference, ψ is the elasticity of intertemporal substitution (EIS), and γ is the risk aversion.

Savers' problem consists of choosing how much to consume and save in order to maximize their expected discounted utility. They can allocate their portfolios between risk-free deposits issued by financiers and can also hold risky assets. Their optimization problem can be written as

$$\max_{\{c_t,\theta_{s,t}\}} U_t,$$

subject to

$$\begin{aligned} dn_{s,t} &= \left[n_{s,t}r_t - c_t + q_t\theta_{s,t}\left(\mathbb{E}_t\left[dR_{s,t}\right] - r_t\right) + T_t\right]dt + q_t\theta_{s,t}\sigma_{q,t}dW_t, \qquad (2) \\ n_{s,t} &\geq 0, \end{aligned}$$

where $n_{s,t}$ is the savers' net worth, $\theta_{s,t}$ is the holding of the risky asset, and T_t is net transfers received from financiers' profits. Transfers are locally riskless because I assume later that the dividend policy implemented by financiers is so.

Financiers are in charge of managing a financial intermediary firm. They operate this firm by issuing deposits to savers as well as using their own wealth, $n_{f,t}$, but they face financing constraints (described later). Their objective is to manage the financial intermediaries' portfolio, and financiers do not consume.[6] Instead, they pay dividends

[6]The assumption that financiers do not consume is different than in Brunnermeier and Sannikov (2014). Assuming there is perfect consumption insurance between the savers and the financiers simplifies the solution of the model and the analysis of the yield curve, because savers are in charge of pricing consumption across time.

to savers. To avoid financiers growing out of their constraints, I follow Gertler and Kiyotaki (2015) and assume a simple dividend policy in which dividends follow an exogenous Poisson process with intensity λ. After paying dividends, financiers receive a fraction $\overline{x}$ of the economy's total wealth to restart the financial firm.[7] Financiers' problem is to maximize the value of the firm (i.e., the expected discounted value of firms' dividends)—that is,

$$V_{f,t} = \max_{\{\theta_{f,t}\}} \mathbb{E}_t \left[\int_t^{\infty} \frac{m_u}{m_t} \lambda e^{-\lambda(u-t)} n_{f,u} \mathrm{d}u \right], \tag{3}$$

subject to

$$\mathrm{d}n_{f,t} = \left[r_t n_{f,t} + q_t \theta_{f,t} \left(\mathbb{E}_t \left[\mathrm{d}R_{f,t} \right] - r_t \right) \right] \mathrm{d}t + \theta_{f,t} q_t \sigma_{q,t} \mathrm{d}W_t, \tag{4}$$

$$V_{f,t} \geq \kappa \theta_{f,t} q_t, \tag{5}$$

$$n_{f,t} \geq 0,$$

where m_t is savers' marginal utility, defined later, and $\theta_{f,t}$ is financiers' holdings of the risky asset. Financiers face a financing constraint, (5), that can be motivated with a standard agency problem. Specifically, I follow Gertler and Kiyotaki (2015) and assume the value of the financial intermediary firm has to be greater than a fraction of the assets the firm holds. This constraint operates as an endogenous leverage constraint.

I next define a competitive equilibrium.

Definition 1 (Competitive Equilibrium) *A competitive equilibrium is a set of aggregate stochastic processes: prices—q_t, r_t; policy functions for savers $(\theta_{s,t}, c_t)$; policy functions for financiers $(\theta_{f,t})$; the value of the financiers' firm $V_{f,t}$; and the utility of savers U_t—such*

[7] In the appendix, I rationalize $\overline{x}$ as a tax to dividends received by savers as well as a capital requirement to start a financial firm.

that

1. *Given prices,* $(\theta_{s,t}, c_t)$ *solves savers' problem*

2. *Given prices,* $(\theta_{f,t}, V_{f,t})$ *solves financiers' problem*

3. *Markets clear (long-term asset, consumption good, and short-term debt):*

$$\begin{aligned}\theta_{s,t} + \theta_{f,t} &= 1, \\ c_t &= \omega\theta_{s,t}Y_t + \theta_{f,t}Y_t, \\ n_{f,t} + n_{s,t} &= q_t,\end{aligned}$$

where the last equation (market clearing for short-term debt) is redundant due to Walras's Law but is useful to explicitly show that wealth holdings add up to total wealth q_t.

The market clearing condition for the goods market, which shows that consumption must be equal to aggregate dividends, is crucial for understanding the results. When savers hold risky assets, $\theta_{s,t} > 0$, aggregate consumption falls because there are deadweight losses associated with savers handling risky assets. One interpretation of the assumption about ω is that financiers lend resources to firms that are more productive than savers when producing consumption goods, as in Brunnermeier and Sannikov (2014). An alternative interpretation is that savers need to pay a cost when holding risky assets, which captures savers' lack of expertise relative to financiers in screening and monitoring investment projects, as in Gertler and Kiyotaki (2015). I provide further discussion in the appendix about the quantitative implications of $\omega < 1$ for consumption and the price of risky capital, and I contrast those implications with the evidence.

Before turning to the solution of the model, it is useful to characterize agents' optimization problems with their first-order conditions. For savers, denoting m_t as their stochastic discount factor, we have

$$r_t = -\mathbb{E}_t\left[\frac{\mathrm{d}m_t}{m_t}\right],$$

and

$$\mathbb{E}_t\left[\mathrm{d}R_{s,t}\right] - r_t\mathrm{d}t \leq -\mathbb{E}_t\left[\frac{\mathrm{d}m_t}{m_t}\mathrm{d}R_{s,t}\right], \tag{6}$$

with equality if households are holding long-term assets (i.e., $\theta_{s,t} > 0$). The savers' stochastic discount factor, as noted in Duffie and Epstein (1992b), is given by

$$\frac{\mathrm{d}m_t}{m_t} = \frac{\mathrm{d}f_{c,t}}{f_{c,t}} + f_{U,t}\mathrm{d}t,$$

where f_c and f_U are the partial derivative of the aggregator (1) with respect to c and U, respectively.

The optimality conditions for financiers require a few more steps. First, notice that because financiers' objective function and constraints are linear in wealth, their value function can be written as

$$V_{f,t} = \phi_t n_{f,t}, \tag{7}$$

where $\phi_t \geq 1$ can be interpreted as financiers' marginal value of wealth (as well as a "Tobin's q").[8] Notice that ϕ_t is an endogenous Itô process whose drift $\mu_{\phi,t}$ and diffusion

[8]See Gertler and Kiyotaki (2015).

$\sigma_{\phi,t}$ are solved in equilibrium. Then the financing constraint can be written as

$$\begin{aligned} \phi_t n_{f,t} &\geq \kappa \theta_{f,t} q_t, \\ \phi_t &\geq \kappa \frac{\theta_{f,t} q_t}{n_{f,t}} \equiv \kappa \alpha_{f,t}, \end{aligned} \tag{8}$$

where $\alpha_{f,t}$ is the endogenous financiers' portfolio share in the risky asset. Financiers' problem can be written as

$$0 = \max_{\theta_{f,t}} \lambda \left(n_{f,t} - V_{f,t} \right) m_t \mathrm{d}t + \mathbb{E}_t \left[\mathrm{d} \left(m_t V_{f,t} \right) \right] + \chi_t \left(V_{f,t} - \kappa \theta_{f,t} q_t \right) \mathrm{d}t, \tag{9}$$

where χ_t is the Lagrange multiplier associated with the financing constraint. Using (7) and (9), the first-order conditions for financiers can be written as

$$\mathbb{E}_t \left[\mathrm{d}R_{f,t} \right] - r_t \mathrm{d}t \geq -\mathbb{E}_t \left[\left(\frac{\mathrm{d}m_t}{m_t} + \frac{\mathrm{d}\phi_t}{\phi_t} \right) \mathrm{d}R_{f,t} \right], \tag{10}$$

with equality if $\chi_t = 0$. Put differently, financiers are the marginal investors in long-term risky assets if their constraints are not binding. If financing constraints are binding, then their holdings in risky assets are pinned down by such constraints (i.e., $\phi_t = \kappa \alpha_{f,t}$), and savers are the marginal investors in risky assets.

Real Yield Curve. I next characterize the yield curve in the economy, which consists of the endogenous price vector for real bonds denoted by $\left\{ P_t^{(\tau)} \right\}_{\tau \geq 0}$, where τ represents the time to maturity of the bond. Yields can then be obtained simply as $y_t^{(\tau)} = -\log P_t^{(\tau)} / \tau$. I assume that a saver is the marginal investor in long-term zero-coupon real bonds. Intuitively, a saver is the marginal investor in the risk-free deposit, which is the relative price of a unit of consumption in the present versus the next instant. Hence, it is natural to assume that a saver is also the marginal agent when pricing long-term bonds, which are the relative price of a unit of consumption in the present versus the near future. The real bond price with time to maturity τ is given by

$$P_t^{(\tau)} = E_t \left[\int_t^{t+\tau} \frac{m_u}{m_t} \mathrm{d}u \right],$$

which can be written as

$$\mathbb{E}_t \left[\frac{\mathrm{d}P_t^{(\tau)}}{P_t^{(\tau)}} \right] - r_t \mathrm{d}t = -cov_t \left(\frac{\mathrm{d}m_t}{m_t}, \frac{\mathrm{d}P_t^{(\tau)}}{P_t^{(\tau)}} \right).$$

Note that, even though a saver is the marginal investor pricing long-term bonds, financial constraints show up in the yield curve indirectly because, in general equilibrium, they distort the allocation of goods and capital—hence afecting the saver's marginal utility. In other words, financing contraints are relevant not because they impede or distort the pricing of the zero-coupon bonds, but because they affect the equilibrium allocation of goods and risky capital, hence affecting the marginal utility of agents that can price the bonds without frictions.

Importantly, my setup cannot answer whether financiers are relatively more relevant for the dynamics of the term premium in the yield curve or for the dynamics of the risk premium in the risky endowment. As shown in Haddad and Muir (2021), intermediaries risk bearing capacity is relatively more relevant for assets that relatively more intermediated (i.e., those asset classes that are more costly for savers to trade). There are at least three reasons for why my model cannot capture the extent to which financiers' risk bearing capacity matter more for long-term bonds or the risky endowment claim. First, in my setup, there is no exogenous variation to intermediaries risk bearing capacity.[9] Instead, financiers risk bearing capacity fluctuates endogenously after fundamental shocks hit the economy. Second, long-term bonds do not affect the equilbrium allocation, but they simply reflect the intertemporal price of consumption in the economy. Thus, in not possible to make a clean comparison between a cross section of assets that differ in their trading costs—a feature that is critical in rationalizing the results in Haddad and Muir (2021). Finally, the costs associated with trading one asset (the endowment claim, where the trading costs can be captured by ω) affect the other asset (long-term bonds) via general equilibrium forces.

I next extend the analysis to study the nominal yield curve.

Nominal Yield Curve. To compute nominal bond prices, I need to introduce money in the analysis. For this, I follow an extensive literature in term structure analysis in endowment economies and assume money is simply a unit of account (i.e., the consumption good is quoted in terms of money) that does not affect the real allocation (see Cox, Ingersoll and Ross (1985b), among others). More precisely, I assume that the inflation rate is pinned down endogenously by a monetary authority that follows an interest rate rule subject to monetary policy shocks. In short, the setup I propose is akin to a two-equation New Keynesian model, because I assume there is no Phillips curve.[10] For this setup, I first define the nominal stochastic discount factor as

$$m_t^{\$} = \frac{m_t}{p_t}, \tag{11}$$

where p_t is the price level (i.e., the price of one unit of the consumption good in terms of money). I assume the price level fluctuates smoothly (i.e., is not affected by Brownian

[9] This exogenous variation could come from shocks to the tighteness of financiers constraint, for example.

[10] See Galí (2015), chapter 2.

shocks),[11]

$$\frac{\mathrm{d}p_t}{p_t} = \pi_t \mathrm{d}t,$$

so that π_t is the inflation rate. I assume the price-level dynamics are pinned down by a central bank following a Taylor Rule. That is, the central bank sets a nominal interest rate i_t^{CB} as

$$i_t^{CB} = \delta_0 + \delta_\pi(\pi_t - \overline{\pi}) + s_t, \tag{12}$$

where s_t is a persistent random variable capturing monetary policy surprises, δ_π is the so-called Taylor loading on inflation, and $\overline{\pi}$ is an inflation target (which can be absorbed by the constant δ_0). The monetary policy surprises follow

$$\mathrm{d}s_t = -\lambda_s s_t \mathrm{d}t + \sigma_s \mathrm{d}W_{s,t},$$

where $W_{s,t}$ is a monetary policy shock uncorrelated with the endowment shock. Then, in equilibrium, the i_t^{CB} must equal the nominal interest rate priced by savers

$$i_t^{CB}\mathrm{d}t = -\mathbb{E}_t\left[\frac{\mathrm{d}m_t^{\$}}{m_t^{\$}}\right]. \tag{13}$$

Using Itô's lemma on (11) and replacing in (13), I obtain an endogenous inflation process,

$$\pi_t = \frac{r_t - s_t - \widehat{\delta}_0}{\delta_\pi - 1},$$

[11] Implicitly, the assumption is that the monetary authority has the tools to pin down a smooth price level; see Di Tella and Kurlat (2021).

with $\widehat{\delta}_0 = \delta_0 - \delta_\pi \overline{\pi}$. Then, the price of a nominal bond with time to maturity τ is

$$P_t^{\$,(\tau)} = \mathbb{E}_t \left[\int_t^{t+\tau} \frac{m_u^\$}{m_t^\$} \mathrm{d}u \right].$$

2 Model Solution

I use the homogeneity property of objective functions and constraints to solve the equilibrium in a recursive fashion, employing a single endogenous state variable,

$$x_t = \frac{n_{f,t}}{q_t} \in [0,1]. \tag{14}$$

The endogenous state variable, x_t, is the net worth of financiers as a share of total wealth in the economy, and it captures how well capitalized financiers are. In particular, low values of x_t represent states in which financiers are more constrained and savers are holding a positive amount of the risky asset, which causes lower levels of consumption because of the inefficiencies associated with savers handling risky assets.

The model's solution consists of solving the endogenous variables in a Markov equilibrium in x_t. I next characterize the equilibrium as a system of ordinary differential equations (ODEs) using the optimality conditions for savers and financiers as well as the market clearing conditions defined in definition 1. For any endogenous variable $z(x)$, I denote its drift and diffusion by $\mu_{z,t}$ and $\sigma_{z,t}$, respectively.

Proposition 1 *The Markov equilibrium is characterized by the following system of ODEs. When the financing constraint (8) is slack (i.e., $\forall x \geq x^*$ where $\frac{\phi(x)}{\kappa} > \alpha(x)$),*

$$
\begin{aligned}
0 &= \frac{1}{p(x)} + \mu_p(x) + \mu + \sigma_p(x)\sigma - r(x) + \left(\sigma_\phi(x) - \gamma\sigma_c(x) + \left(\frac{1}{\psi} - \gamma\right)\sigma_\xi(x)\right)(\sigma_p(x) + \sigma), \\
0 &= \frac{\lambda(1-\phi(x))}{\phi(x)} + \mu_\phi(x) - \left(\gamma\sigma_c(x) + \left(\frac{1}{\psi} - \gamma\right)\sigma_\xi(x)\right)\sigma_\phi(x), \\
0 &= \frac{\rho}{1-\frac{1}{\psi}}\left\{\xi(x)^{\frac{1}{\psi}-1} - 1\right\} + \mu - \frac{\gamma}{2}\sigma^2 + \mu_\xi(x) - \frac{\gamma}{2}\sigma_\xi(x)^2 + (1-\gamma)\sigma_\xi(x)\sigma.
\end{aligned}
$$

When the financing constraint (8) is binding (i.e., $\forall x < x^*$, where $\frac{\phi(x)}{\kappa} = \alpha(x)$),

$$
\begin{aligned}
0 &= \frac{\omega}{p(x)} + \mu_p(x) + \mu + \sigma_p(x)\sigma - r(x) + \left(\left(\frac{1}{\psi} - \gamma\right)\sigma_\xi(x) - \gamma\sigma_c(x)\right)(\sigma_p(x) + \sigma), \\
0 &= \frac{\lambda(1-\phi(x))}{\phi(x)} + \frac{\phi(x)}{\kappa}\left(\frac{(1-\omega)}{p(x)} + \sigma_\phi(x)(\sigma_p(x)+\sigma)\right) + \mu_\phi(x) - \left(\gamma\sigma_c(x) + \left(\frac{1}{\psi} - \gamma\right)\sigma_\xi(x)\right)\sigma_\phi(x), \\
0 &= \frac{\rho}{1-\frac{1}{\psi}}\left\{\xi(x)^{\frac{1}{\psi}-1} - 1\right\} + \mu_c(x) - \frac{\gamma}{2}\sigma_c(x)^2 + \mu_\xi(x) - \frac{\gamma}{2}\sigma_\xi(x)^2 + (1-\gamma)\sigma_\xi\sigma_c(x).
\end{aligned}
$$

The real interest rate is $r(x_t) = -\mathbb{E}_t\left[\frac{dm_t}{m_t}\right]$, where $m_t = \exp\left(\int_0^t f_U du\right) f_c$, real bonds are $P(x,\tau) = \mathbb{E}_t\left[\int_t^\tau \frac{m_u}{m_t} du\right]$, and nominal bonds are $P^\$(x,\tau) = \mathbb{E}_t\left[\int_t^\tau \frac{m_u^\$}{m_t^\$} du\right]$, solving

$$
\begin{aligned}
0 &= -\frac{P_\tau(x,\tau)}{P(x,\tau)} + \mu_P(x,\tau) + \frac{1}{2}\sigma_P(x,\tau)^2 - r(x) - \left(\gamma\sigma_c(x) + \left(\frac{1}{\psi} - \gamma\right)\sigma_\xi(x)\right)\sigma_P(x,\tau), \\
0 &= -\frac{P_\tau^\$(x,s,\tau)}{P^\$(x,s,\tau)} + \mu_P^\$(x,s,\tau) + \frac{1}{2}\sigma_P^\$(x,s,\tau)^2 - i(x,s) - \left(\gamma\sigma_c(x) + \left(\frac{1}{\psi} - \gamma\right)\sigma_\xi(x)\right)\sigma_{P^\$}(x,s,\tau),
\end{aligned}
$$

with $P(x,0) = 1\ \forall x$ and $P^\$(x,s,0) = 1,\ \forall (x,s)$. The terms $\mu_p(x)$, $\mu_\phi(x)$, $\mu_\xi(x)$, $\mu_c(x)$, $\mu_P(x,\tau)$, $\mu_P^\$(x,s,\tau)$, $\sigma_p(x)$, $\sigma_\phi(x)$, $\sigma_\xi(x)$, $\sigma_c(x)$, $\sigma_P(x,\tau)$, and $\sigma_{P^\$}(x,s,\tau)$ are partial derivatives obtained by applying Itô's lemma in their corresponding functions. The law of motion for x, as well as additional boundary conditions, is shown in the appendix.

Proof. See appendix.

The aforementioned proposition shows the system of ODEs that solves for financiers' marginal value $\phi(x_t)$; the price-dividend ratio $p(x_t) = q_t/Y_t$; savers' value function $U(x,c)$; and real and nominal bond prices. The first three equations show the conditions when the financing constraint is slack. In such a region of the state space, the risky asset is held only by financiers (i.e., $\theta_{f,t} = 1$). The first expression shows the pricing equation for the risky asset, coming from financiers' first-order condition. The second expression pins down financiers' marginal value of wealth, ϕ_t, which is coming from introducing the first-order conditions into financiers' value function (9). The third expression solves for savers' value function, using the transformation $U_t = (c_t \xi(x_t))^{(1-\gamma)}/(1-\gamma)$ and solving for $\xi(x_t)$.

The second set of equations shows the system when financiers' financing constraint is binding (so savers hold a fraction of the risky asset). The fourth equation shows the equilibrium condition for the risky asset, the fifth equation shows financiers' value function, and the sixth equation shows savers' value function (using the transformation mentioned in the previous paragraph). Finally, the last two equations are the pricing formulas for real and nominal bond prices, respectively.

Importantly, the point in the state space in which the constraint becomes binding, x^*, is characterized as follows. When x is high, financiers are well capitalized, their financing constraint is slack, and, as a result, they hold the risky asset using a moderate amount of leverage. That is, when $x_t > x^*$, we have $\phi_t > \kappa\alpha_{f,t} = \kappa/x_t$, where the last equality holds by the market clearing condition in the risky asset. As x_t declines, financiers' leverage increases, and excess returns increase, until a point, x^*, in which the constraint starts to bind and savers hold a positive amount of the risky asset.[12]

[12] I provide more details about how to find x^* in the appendix, where I describe the numerical algorithm to solve the model.

3 Results

Calibration. I calibrate the model at an annual frequency and solve it using a global solution technique based on projection methods. I provide a detailed description of the solution method in the appendix. Table 1 shows the three groups of parameters—namely, Preferences and Endowment, Financiers, and Nominal Rate. First, for Preferences and Endowment, as I highlight later, the risk aversion (γ) and the EIS ($1/\psi$) play a critical role in pinning down the dynamics of the real interest rate and the yield curve. This is because γ and ψ control the relative strength of the precautionary savings (i.e., changes in the interest rate followed by changes in the volatility of consumption) and intertemporal substitution (i.e., changes in the interest rate caused by changes in the expected change of consumption) forces. In the baseline calibration, I set $\gamma = 5$ and $\psi = 1/5$, which means the savers have CRRA (constant relative risk aversion) preferences. I later relax this assumption and solve the model with alternative preference parametrization and explain why the EIS plays a critical role in the results. For the endowment, I set $\sigma = 0.036$, a number that is in line with the volatility of productivity in the US and close to the value used in He and Krishnamurthy (2019).[13] Lastly, I set $\mu = 0.007$ to match the average level of the real rate.

Second, for financiers' technology and constraint, I calibrate $\kappa = 0.4$ to target an average leverage of 3 (He and Krishnamurthy (2019)). I set $\lambda = 0.08$, which gives an expected payout rate of the intermediary as in Gertler and Kiyotaki (2015). The parameter $\overline{x}$ can be interpreted as a tax rate on dividends received by savers; hence, I set $\overline{x} = 0.2$ as

[13] Previous papers have used a wide range of values for σ. For example, He and Krishnamurthy (2013), in a similar setup, use $\sigma = 0.09$; Brunnermeier and Sannikov (2014), also in a similar setup but with endogenous production, use $\sigma = 0.1$.

a plausible tax rate.[14] Lastly, ω is a crucial parameter, and I set ω=0.85. From a quantitative point of view, ω=0.85 implies that the price-dividend ratio and consumption can drop, at most, 50 percent and 15 percent, respectively (this would be the case when intermediaries have no wealth and savers hold the entire wealth in the economy). Importantly, in equilibrium, households will almost surely never hold the entire wealth in the economy (i.e., the probability of x reaching zero is almost surely zero). Indeed, only in very rare occasionsy, the price-dividend ratio and consumption will drop more than 25 percent and 7 percent, respectively, in a given year.[15]

Finally, for the monetary policy rule, I set $\delta_\pi = 1.5$, which is a commonly used parametrization since Taylor (1993) and broadly consistent with the evidence. I calibrate the monetary policy shock to match the persistence and volatility of the surprises documented in Gertler and Karadi (2015).

Solution and Mechanism. Figure 1 shows the solution of the key endogenous variables. All panels display the endogenous variables in the Markov equilibrium—that is, as a function of the state variable x. The red dashed line in all panels represents the point at which the financing constraint binds.

The invariant distribution, displayed in the lower-left panel of Figure 1, shows the economy has two modes. It spends the majority of the time in a normal regime, where constraints are slack (i.e., to the right of the red dashed line), and some time in a crisis regime, where constraints are binding. Normal times are characterized by low volatility, low rates, and moderate leverage. As is common in these types of models, leverage

[14] The sole purpose of $\overline{x}$ is to avoid a degenerate distribution of wealth and it does not affect the qualitative results—it does not affect marginal decisions. In the appendix, I illustrate how $\overline{x}$ can be interpreted as a tax rate.

[15] I discuss further details about ω in the appendix and show that the equilibrium-implied losses in consumption and asset prices are broadly in line with the empirical evidence documented in Muir (2017) and Greenwood, Hanson, Shleifer and Sørensen (2022).

is countercyclical: The lower the intermediaries' wealth (i.e., lower x), the higher the leverage (as shown by the lower-right panel).

If the economy is in the normal regime and a sufficiently negative aggregate shock occurs, financial intermediaries reallocate their portfolios, the price of risky assets declines, and the price of risk increases (upper-right panel). Financing constraints may bind (depending on the magnitude of the shock) and trigger the well-known financial accelerator mechanism studied in previous literature (e.g., Bernanke, Gertler and Gilchrist (1999)), in which lower valuations deteriorate intermediaries' wealth even further. Following the first-order conditions (6) and (10), the price of risk when financiers are constrained is given by

$$-\mathbb{E}_t\left[\frac{\mathrm{d}m_t}{m_t}\mathrm{d}W_t\right]/\mathrm{d}t = \gamma\sigma_{c,t} - \left(\frac{1}{\psi}-\gamma\right)\sigma_{\xi,t},$$

and, when financiers are unconstrained, is given by

$$-\mathbb{E}_t\left[\left(\frac{\mathrm{d}m_t}{m_t}+\frac{\mathrm{d}\phi_t}{\phi_t}\right)\mathrm{d}W_t\right]/\mathrm{d}t = \gamma\sigma_{c,t} - \left(\frac{1}{\psi}-\gamma\right)\sigma_{\xi,t} - \sigma_{\phi}.$$

The increase in the price of risk when financiers are unconstrained is primarily due to two reasons. First, financiers are leveraged, and such leverage increases when x declines. As a result, the volatility of x increases when x declines.[16] Second, financiers marginal utility of wealth, ϕ_t, increases when x declines. Hence, the diffusion component associated with ϕ_t, $\sigma_\phi = \frac{\phi_x}{\phi}x\sigma_x$, is negative and increases with σ_x. When the constraint binds, the price of risk is dominated by the increase in the volatility of consumption, σ_c. As x declines further, the amplification role of x decreases and endogenous

[16] I show that the volatility of x, σ_x, is proportional to financiers' leverage in the appendix.

volatility declines, hence causing the volatility of x and, as a result, the price of risk to decline.[17]

A central element in the yield curve dynamics is the behavior of the short-term interest rate, r, shown in the upper-left panel. Notice that when the economy enters a crisis regime, the price of risk spikes, and the real interest rate increases. This is because wealth is transferred to savers, who are inefficient in handling risky assets, which means the level of aggregate dividends (and consumption) declines. Because the inefficiencies caused by the misallocation of risky assets are temporary, savers expect the consumption level to increase in the future, which causes an increase in the real interest rate. Put differently, the dynamics for the consumption level are similar to a random walk with drift, where deviations from the trend are persistent. When consumption is below the trend, it is expected to mean-revert in the future. In the model, the trend is endogenously driven by financial intermediaries' wealth dynamics.

The Real Yield Curve. Intuitively, investors require a premium to hold an asset whose value persistently declines in states in which the price of risk is high. This is precisely what drives the real term premium in the economy: Real bond prices decline (i.e., real rate persistently increases) in states in which the price of risk is high. Figure 2 shows the average yield curve in the economy. Simply put, the presence of an unconditionally positive term premium causes the yield curve to be upward sloping on average. The left panel of Figure 2 illustrates the dynamics of yields at different horizons across the state space. The mechanism through which financial intermediaries reduce their positions in risky assets by selling those to less efficient savers is more pronounced in short-maturity

[17] In the baseline calibration, I set $\frac{1}{\psi} = \gamma$; hence the term $\left(\frac{1}{\psi} - \gamma\right) \sigma_{\xi,t}$ is equal to zero. Qualitative results do not change when $\frac{1}{\psi} \neq \gamma$—the price of risk displays a similar behavior across x to that when $\frac{1}{\psi} = \gamma$, as the term $\sigma_{\xi,t}$ is small relative to $\sigma_{c,t}$.

rates—long-term yields are less sensitive to the misallocation of wealth in the economy. Put differently, current fluctuations in financiers' wealth have a lower incidence in driving longer-maturity bonds, a feature that can be appreciated in the left panel of Figure 2. The panel shows the yield of bonds at 1-, 10-, and 30-year maturities and displays the yield of a very long-term bond. As the horizon of the bond increases, the yields become less sensitive to current financial conditions, and x_t has a smaller effect on yields' dynamics. This result, driven by the endogenous persistence of x, shows that even very long-term rates can display substantial volatility.

The upper panel of Figure 3 shows the real yield curve for different levels of x. The circles in the figure represent the average real yields.[18] The yield curve is approximately flat when x is high, mainly because term premia and real rates are low. When x is high, financiers are relatively well capitalized, and financing constraints are slack. When x is low, however, the economy is in crisis times, constraints are binding, and real yields are high. In this state, the short-term rate is expected to mean-revert, and this force pushes down long-term rates (even though term premia are high, the expectations of the short rate dominate). Thus, the gray line shows a downward-sloping yield curve. Finally, the lower panel of Figure 3 shows the standard deviations of yields across maturities. As can be seen, the model can capture well the first and second moments of the real yield curve.

The Nominal Yield Curve. Figure 4 shows the nominal yield curve in the baseline calibration. The top panel displays the nominal yield curve when x is at its mean value and shows the yield curve at different values for the other state variable, the monetary pol-

[18] I use both data for Treasury Inflation Protected Securities (TIPS) in the period 2002:Q1 to 2018:Q4 and data from Chernov and Mueller (2012) for the period 1971:Q3 to 2001:Q4. I explain in the appendix all the details about the data used in the paper.

icy shock s_t. In equilibrium, a low s_t translates into a higher inflation, which ultimately causes higher nominal interest rates. This result can be seen in the equilibrium nominal short rate, which is the result of plugging the endogenous inflation process (1) into the Taylor rule (12):

$$i_t = \widetilde{\delta}_0 + \left(\frac{\delta_\pi}{\delta_\pi - 1}\right) r_t + \left(\frac{1}{1 - \delta_\pi}\right) s_t, \tag{15}$$

where $\widetilde{\delta}_0$ is an adjusted constant similar to δ_0.[19] Then, as long as $\delta_\pi > 1$ (which is the empirically and theoretically relevant case), a higher (lower) s_t will, in equilibrium, cause lower (higher) nominal rates because of the endogenous response in inflation. The lower panel of Figure 4 shows the nominal yield curve for different values of x, holding s_t at its unconditional mean. As noted in Schneider (2022), $\delta_\pi > 1$ produces a nominal term premium that is larger than the real term premium. Hence, the model can capture the evidence that the slope of the nominal yield curve is, on average, approximately twice as big as the slope of the real yield curve. Intuitively, this is because, in order to pin down inflation, the monetary authority must adjust the nominal interest rate to changes in the state of the economy in a relatively stronger fashion than the adjustment of the real rate.

Precautionary Savings and Intertemporal Substitution. The dynamics of the short-term interest rate are driven by the interplay of the consumption dynamics and the preferences parameters—the risk aversion and EIS.[20] This point can be clearly seen in the case in which the economy is at the first best and financial frictions do not affect the

[19] $\widetilde{\delta}_0 = \delta_0 - \delta_\pi \overline{\pi} - \frac{\delta_\pi}{\delta_\pi - 1} \widehat{\delta}_0$.

[20] Another possible force that could cause changes in the real interest rate is a non-neutral monetary policy.

allocation. In such a case, the interest rate is given by[21]

$$r = \rho + \underbrace{\frac{\mu}{\psi}}_{IS} - \underbrace{\frac{1}{2}\left(1+\frac{1}{\psi}\right)\gamma\sigma^2}_{PS}. \tag{16}$$

I denote the second term in (16) as the intertemporal substitution (IS) motives and the third term as the precautionary savings (PS) motives (see, for example, Kimball and Weill (2009)). The IS motives relate the expected consumption growth to the interest rate, depending on the magnitude of the EIS, ψ. An increase in the expected consumption growth, μ, increases the interest rate, and this effect is stronger when the EIS is lower. That is, when savers have a lower EIS, the interest rate has to increase relatively more if μ is higher in order to incentivize savers to smooth consumption over time and clear the goods market. The PS motives relate the conditional volatility of consumption growth to the interest rate, depending on the magnitude of risk aversion (for a given EIS). An increase in the uncertainty of consumption growth, a higher σ, reduces the lever of rates as risk averse savers seek to save more when they face a more uncertain consumption path.

In the model with financial frictions, the intuition is similar to the one in the frictionless case. An increase in the expected consumption growth pushes the short rate higher through the IS channel, while an increase in the conditional volatility of consumption pushes the short rate lower through the PS one. The key difference is that the short rate cannot be solved in closed form as in expression (16). Instead, the short rate and the consumption dynamics depend on the endogenous state variable, x (i.e., $\mu_c(x)$ and $\sigma_c(x)$). As a result, both the PS and IS motives would be a function of x. When

[21] See the appendix for the derivation of equation (16).

the financial friction is binding and savers are holding risky assets, both expected consumption growth and its volatility increase. On net, if the interest rate declines when the constraint binds, this means the $PS(x)$ must be dominating the $IS(x)$ motives. I next study the net effect of the $PS(x)$ and $IS(x)$ on the short rate and its effect on the slope of the yield curve.

Figure (6) shows the dynamics of the short rate (left panel) as well as the slope of the real yield curve (right panel) for different combinations of risk aversion and EIS. The baseline calibration ($\gamma = 5$ and $\psi = 1/5$) is shown in blue. I label this calibration as "High risk aversion, low low EIS." In red, I show the results with a higher EIS but the same risk aversion as in the baseline ($\gamma = 5$ and $\psi = 1/3$). This calibration can be labeled as "High risk aversion, high EIS." As shown, increasing the EIS —while holding the risk aversion at the baseline level—reduces the term spread to approximately one-third of the term spread obtained in the baseline calibration. The reduction in the term spread is because a higher EIS increases the relative importance of the PS motives over the IS motives when the constraint binds. As the volatility of consumption increases, the short rate declines, as shown in the left panel. That is, the net effect of the PS and the IS forces is that the PS motives dominate over the IS. The decline in the short rate implies that real bond prices increase at some point during bad times, hence pushing down the average real term premium. As the wealth of intermediaries declines further, the pricing effect of intermediaries declines so that volatility decreases. Then the IS motives dominate again, and the real rate increases as the consumption level declines further when x goes to very low levels.

In yellow, I show the results with a higher EIS and lower risk aversion than in the baseline ($\gamma = 3$ and $\psi = 1/3$)—a "Low risk aversion, high EIS" calibration. The results are similar to the case in which only the EIS is higher than in the baseline, but with a somewhat weaker effect of the PS motives. The short rate declines somewhat when the financing constraint binds, which indicates that the PS motives are stronger than the IS motives, but less so than in the case with $\gamma = 5, \psi = 1/3$. Notice that the case with $\gamma = 3, \psi = 1/3$, implies a similar term spread to that in the case with $\gamma = 5, \psi = 1/3$. This result indicates that the vast majority of the decline in the term spread from the baseline calibration, "High risk aversion, low EIS", to the "Low risk aversion, high EIS" calibration can be attributed to the EIS rather than the risk-aversion coefficient.

Finally, in gray, I show the results with the basline risk aversion but an EIS greater than 1 ($\gamma = 5$ and $\psi = 6/5$)—a calibration with "High risk aversion and EIS greater than 1." Results show a similar intuition to those in the previous cases. A higher EIS translates into a smaller IS force relative to the PS motives when the constraint binds. As a result, the short rate declines relatively more than in the other cases, reducing the average term spread.

Time-Varying Term Premium and Bond Return Predictability. A central property of the model is that expected excess returns on bonds are time varying. As a consequence, long-term yields fluctuate not only because of changes in the expected path of short-term rates, but also because of changes in the term premium. In other words, the so-called expectations hypothesis is rejected in the model, a feature that is consistent with a large body of evidence.[22]

The upper-left panel in Figure (5) shows the dynamics of the term premium in the model, for different maturities, across the state space. The average term premium is positive and increasing across maturities, as shown by the upper-right panel of Figure

[22] Duffee (2013) provides a summary.

(5). Intuitively, bonds of longer maturity contain more interest rate risk than do bonds of short maturity and, as a consequence, carry a larger premium. Notice that the term premium spikes when the constraints bind, as the price of risk increases and real bond prices decline, in a similar dynamic to the one explained in the mechanism in the previous subsection.

Because the term premium fluctuates over time, a natural question to ask is whether information in the yield curve at time t helps in predicting future fluctuations in term premia. In particular, the seminal work of Fama and Bliss (1987) shows that the forward-spot spread predicts future excess returns on bonds, which is one of the salient properties of the empirical evidence about the yield curve. I next study the extent to which the predictability of bond returns is captured by the model. For this exercise, I conduct a predictability analysis following Fama and Bliss (1987). I run, using simulated data from the model, the following regressions:

$$rx_{t+1}^{(\tau)} = \alpha^{(\tau)} + \beta^{(\tau)} \left(f_t^{\$(\tau)} - y_t^{\$(1)} \right) + \epsilon_{t+1}, \tag{17}$$

where $rx_{t+1}^{(\tau)} = p_{t+1}^{\$(\tau)} - p_t^{\$(\tau)} - y_t^{\$(1)}$, with $p_t^{\$(\tau)} = \log P_t^{\$(\tau)}$, is the excess returns of a $\tau-$maturity bond; $f_t^{\$(\tau)}$ is the one-year nominal forward rate between maturity n and $n-1$; and $y_t^{\$(1)}$ is the one-year nominal rate.

As noted in Fama and Bliss (1987), a positive $\beta^{(\tau)}$ indicates that the term premium fluctuates through time and that a higher forward-spot spread predicts higher expected excess returns on bonds. Figure (5) shows the results for the $\beta^{(\tau)}$ in the model, for maturities between two and five years (as in Fama and Bliss (1987)). As shown, model-implied coefficients are positive across all maturities, which implies the forward-spot spread in the model contains information about future bond excess returns, consistent with the evidence reported in Fama and Bliss (1987).

A complementary way to illustrate the predictability results in the model is to numerically compute the elements in regression (17) and study how they change across the state space. For this analysis, I study how expected excess returns and the forward-spot spread change across the state space.

The lower-right panel of Figure (5) shows the expected excess returns for a five-year bond as well as the five-year forward-spot spread across the state space. Consistent with the estimates of equation (17) shown in the lower-left panel of Figure (5), as well as the results in Fama and Bliss (1987), the model implies a positive co-movement between the expected excess returns and the forward-spot spread. In other words, movements in long-term one-year forwards (relative to the one spot one-year rate) are primarily driven by the spike in the term premium.

Term Structure of Conditional Distributions. Recent literature has stressed the role of financing conditions when forecasting the distribution of future real variables (Giglio et al. (2016);Adrian et al. (2019)). The term structure of distributions of real variables is related to the yield curve because the former is a forecast of the conditional distribution of a random variable at a certain point in the future, while the latter is the conditional expectation of marginal utility at a certain point in the future.[23] I show the key economic forces driving the yield curve, elaborated earlier, are also consistent with the evidence about the term structure of conditional distributions of future outcomes.

To compute the term-structure of distributions, consider a process z_t in the model that follows an Itô process

$$\mathrm{d}z_t = \mu_{z,t}\mathrm{d}t + \sigma_{z,t}\mathrm{d}W_t,$$

[23] Technically, these two objects are the forward Kolmogorov equation and the backward Kolmogorov equations.

where $\mu_{z,t} = \mu_z(x_t)$ and $\sigma_{z,t} = \sigma_z(x_t)$ are the drift and diffusion, respectively. Next, I define the function $f(x_s|x_t = \tilde{x}, s)$ as the conditional distribution of x at each point in time $s > t$, starting from a point $\tilde{x}$. The evolution of the density over time can be described by the following partial differential equation:

$$\frac{\partial f(z(x)|\tilde{x},t)}{\partial t} = -\frac{\partial}{\partial x}\left[f(z(x)|\tilde{x},t)\,\mu(x)\right] + \frac{1}{2}\frac{\partial^2}{\partial x^2}\left[f(z(x)|\tilde{x})\,\sigma(x)^2\right],$$

which is also known as the forward Kolmogorov equation (or Fokker-Planck equation).

Figure 7 shows the forecasted conditional distributions for consumption (top two panels) and the state variable x (bottom two panels) at different horizons. The blue line represents the forecasted densities conditional on current financial conditions being loose. More precisely, the distributions are forecasted conditional on $x_t = \tilde{x}$, where $\tilde{x}$ is 10 percent above the point at which financing constraints bind. The distribution in red represents the forecasted density conditional on current financial conditions being tight. I assume x_t is 10 percent below the point where financing constraints bind. The red dashed line in all four panels represents the point in the state space at which financing constraints bind.

In line with the evidence reported in Adrian et al. (2019), the top two panels indicate that, conditional on the economy facing tight financial conditions, the distribution of future consumption is negatively skewed. Also, the conditional distribution fluctuates across the horizon, and the relatively negative skewness of the conditionally constrained distribution persists even in the three-year-ahead forecasted distribution. The main source of the asymmetry between the constrained and unconstrained distributions is that economic outcomes are quite different in the constrained and unconstrained regions. For example, in the constrained region, the economy is more leveraged (and thus more sensitive to shocks), the real rate is much more volatile, and the price of risk moves faster. These conditions may persist because it takes time for financiers' wealth to be rebuilt. Simply put, x is a persistent process.

The bottom two panels display the forecasted conditional distributions for the state variable x. The intuition is similar to that of consumption growth. Tight financial conditions are persistent and can trigger quite volatile and unstable outcomes. Simply put, the model rationalizes the data with two main elements: Tighter financial conditions are persistent outcomes, and they lead to quite different economic outcomes than those implied by the economy functioning in an unconstrained region. As with the yield curve, the key elements are the bimodal nature of the economy and the persistent dynamics of intermediaries' wealth.

4 Conclusion

Financial intermediaries hold long-term assets, which means fluctuations in long-term yields affect the extent to which intermediaries are financially constrained. These constraints affect marginal valuations in the economy not only of the intermediaries, but also, in general equilibrium, of other agents. Hence, because long-term yields are forecasts of marginal valuations, financing constraints and long-term yields are directly related to each other.

In this paper, I show that financing constraints generate an endogenously time-varying real term premium that is consistent with the data. The nominal yield curve is primarily driven by the real factor that captures the health on intermediaries' balance sheets. The mechanism I propose can rationalize relevant yield curve facts, such as an upward-sloping real yield curve and highly volatile long-term yields, which are indeed hard to capture in standard macro models (Duffee, 2018). These results are purely driven by the fact that intermediaries' financing constraints may occasionally bind, linking the yield curve to intermediaries' financial health.

The novel economic mechanism I propose, connecting intermediaries' wealth and the yield curve, can rationalize interesting macroeconomic phenomena, suggesting that there are several potential avenues for further research. In particular, I show that the same mechanism explaining the term structure of interest rates is also able to rationalize the negative skewness in the term structure of distributions of consumption when financing constraints are binding.

There are, however, several important quantitative elements of yields that the model cannot rationalize. For example, the model implies a positive correlation between bond and stock returns, while the evidence suggests that the sign of such a correlation has changed from positive to negative in the past few decades (Campbell, Pflueger and Viceira (2020); Chernov, Lochstoer and Song (2023)). Also, the model cannot rationalize the time-variation in yields' conditional skewness, an important property of yields that helps in understanding the time variation of bond returns (Bauer and Chernov (2023)). Indeed, the model implies that conditional skewness is always positive and such conditional skewness has very little power for predicting bond returns. I speculate that incorporating additional state variables—for example, by incorporating heterogeneous intermediaries and savers—could help in addressing some of these issues.

5 Tables and Figures

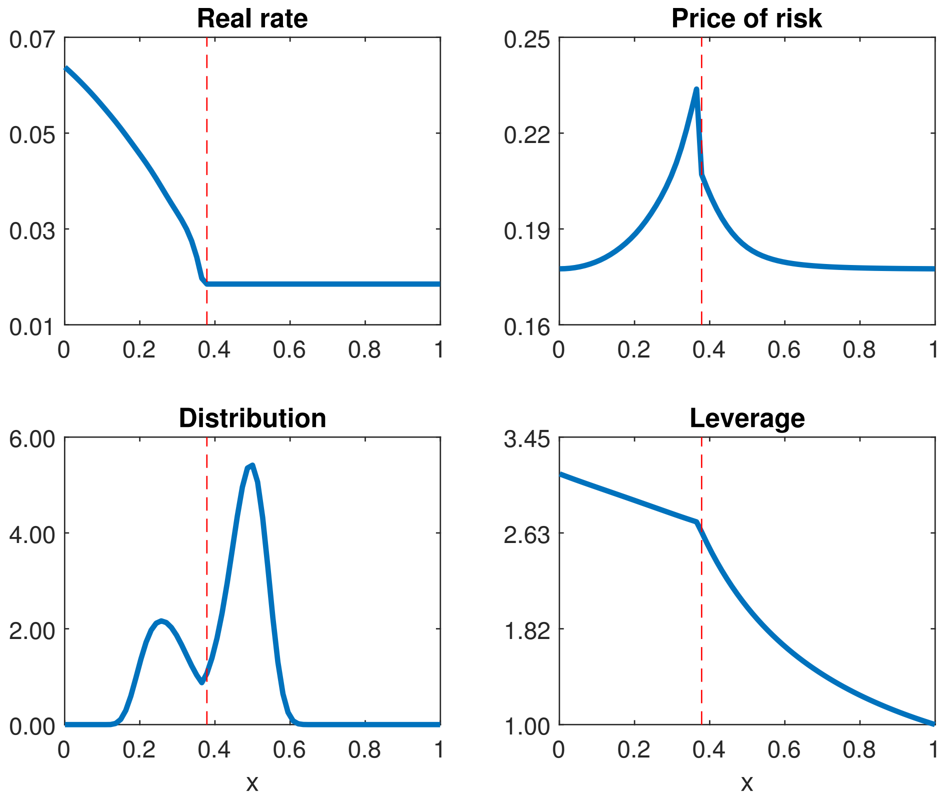
TABLE 1. Calibration

PARAMETERS

		Value	Description
1. Preferences and Endowment			
	γ	5	Risk aversion
	ψ	1/5	EIS
	μ	0.007	Drift Y_t
	σ	0.036	Volatility Y_t
2. Financiers			
	λ	0.08	Dividend payout
	ω	0.85	Management cost
	κ	0.4	Fraction divertible assets
3. Nominal Rate			
	δ_π	1.5	Taylor coefficient
	λ_s	0.06	Persistence of monetary shocks
	σ_s	0.0024	Volatility of monetary shocks

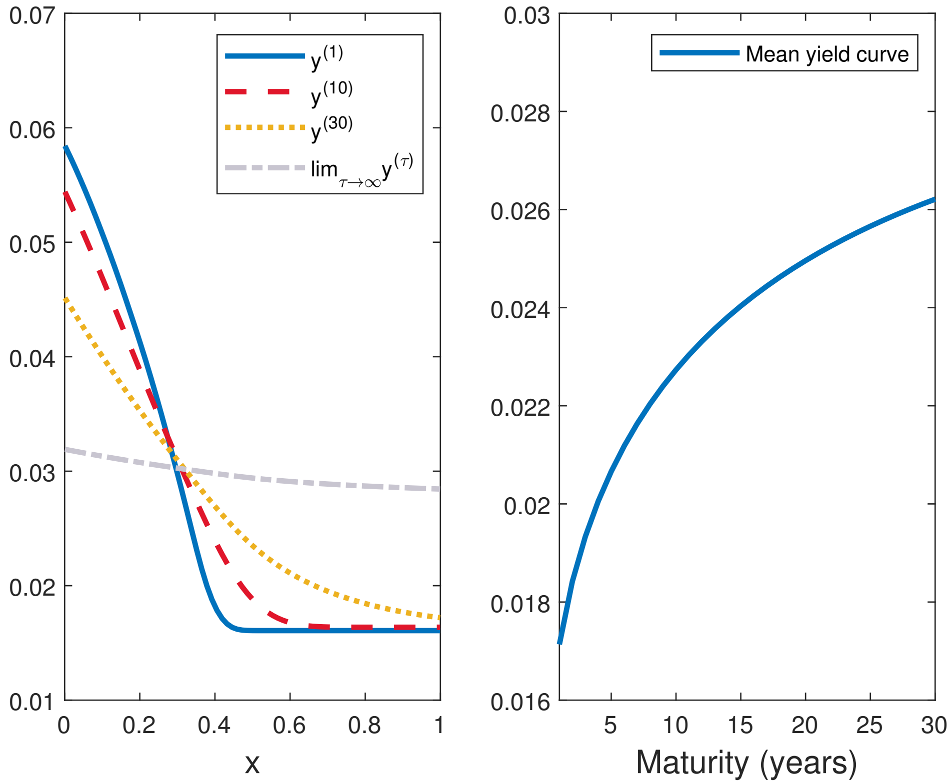
NOTE: This table shows the calibration of the model at an annual frequency.

FIGURE 1. Model Solution



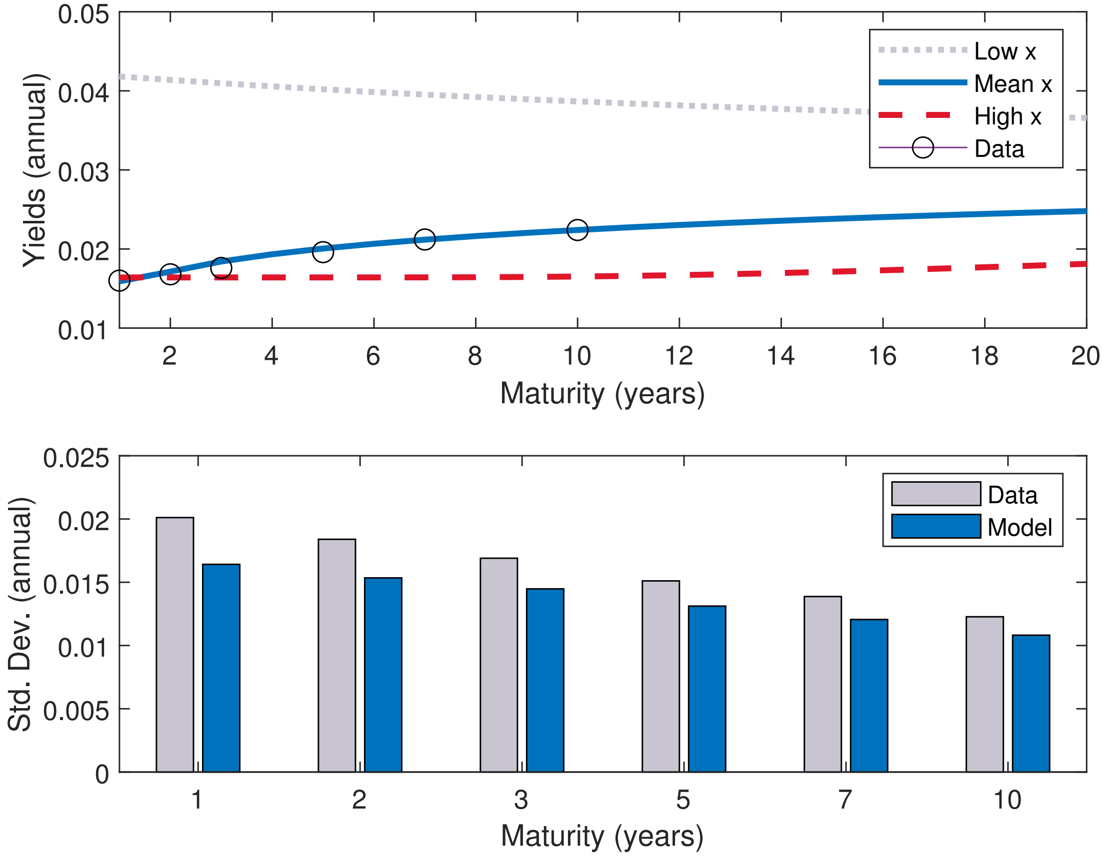
NOTE: This figure shows the model solution, with the calibrated parameters from Table 1. The red dashed line represents the point in the state space at which the financial constraint binds.

FIGURE 2. Model Solution: Yields



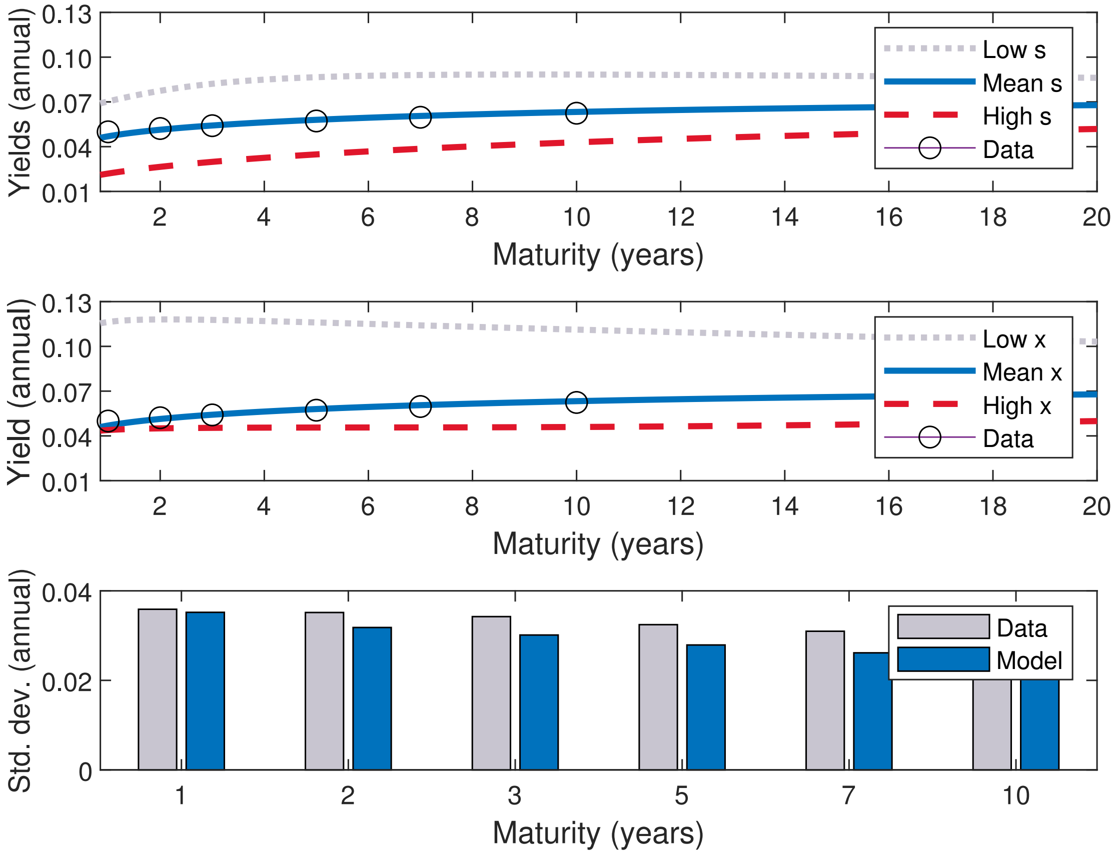
NOTE: This figure shows the real yield curve in the model, with the calibrated parameters from Table 1. The left panel shows yields for different maturities over the state space. The right panel shows the average yields.

FIGURE 3. Real Yield Curve



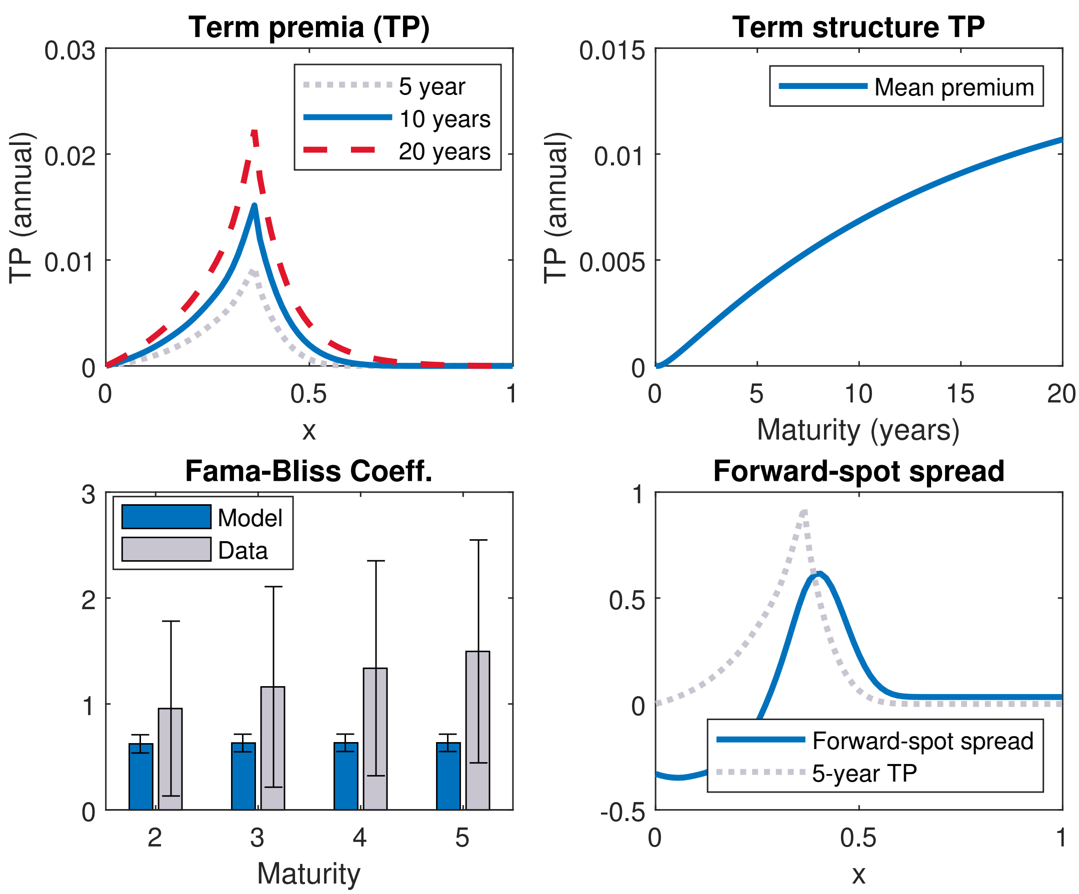
NOTE: This figure shows the real yield curve in the model, with the calibrated parameters from Table 1. The top panel displays the yield curve for three different levels of x (mean, plus two standard deviations, and minus two standard deviations). The bottom panel show the standard deviation of yields. The data for real yields are from Chernov and Mueller (2012) for the 1971-2002 period, and the TIPS data are from Gürkaynak, Sack and Wright (2010) for the 2003-18 period. See the appendix for more details about the data.

FIGURE 4. Nominal Yield Curve



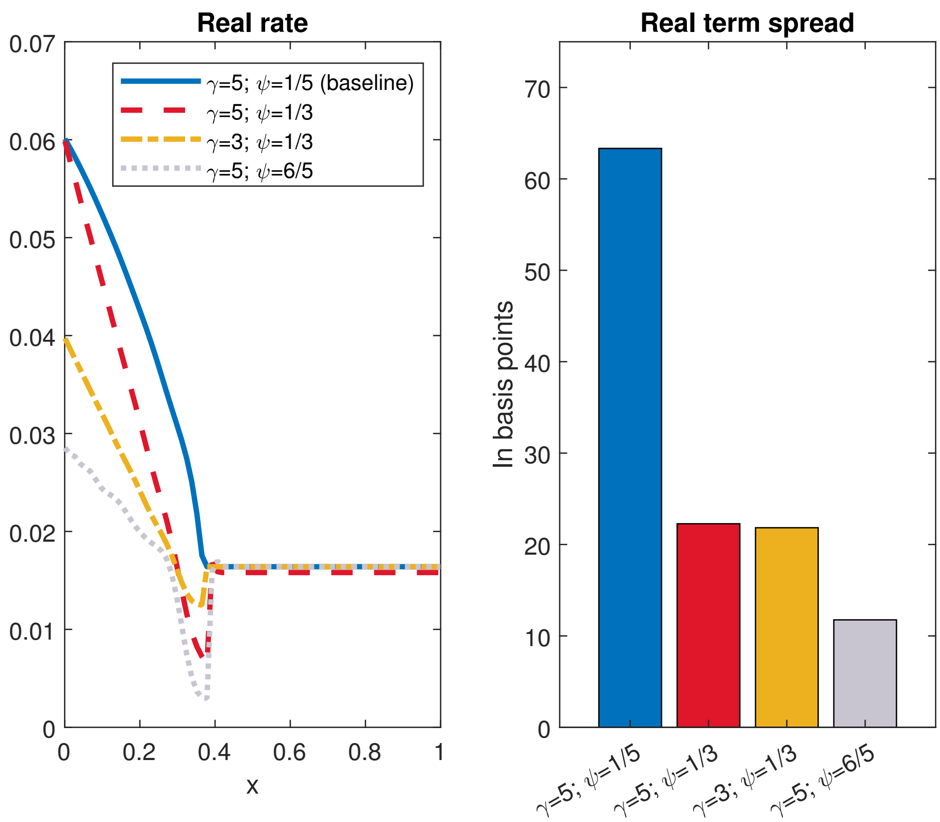
NOTE: This figure shows the nominal yield curve in the model, with the calibrated parameters from Table 1. The top panel displays the yield curve for three different levels of the persistent monetary policy shock variable s (mean, plus two standard deviations, and minus two standard deviations), when x is at its steady state value. The bottom panel displays the yield curve for three different levels of the persistent state variable x (mean, plus two standard deviations, and minus two standard deviations), when s is at its steady state value. The data for nominal yields are from Gürkaynak, Sack and Wright (2007). I provide details about the data in the appendix.

FIGURE 5. Term Premium and Bond Return Predictability



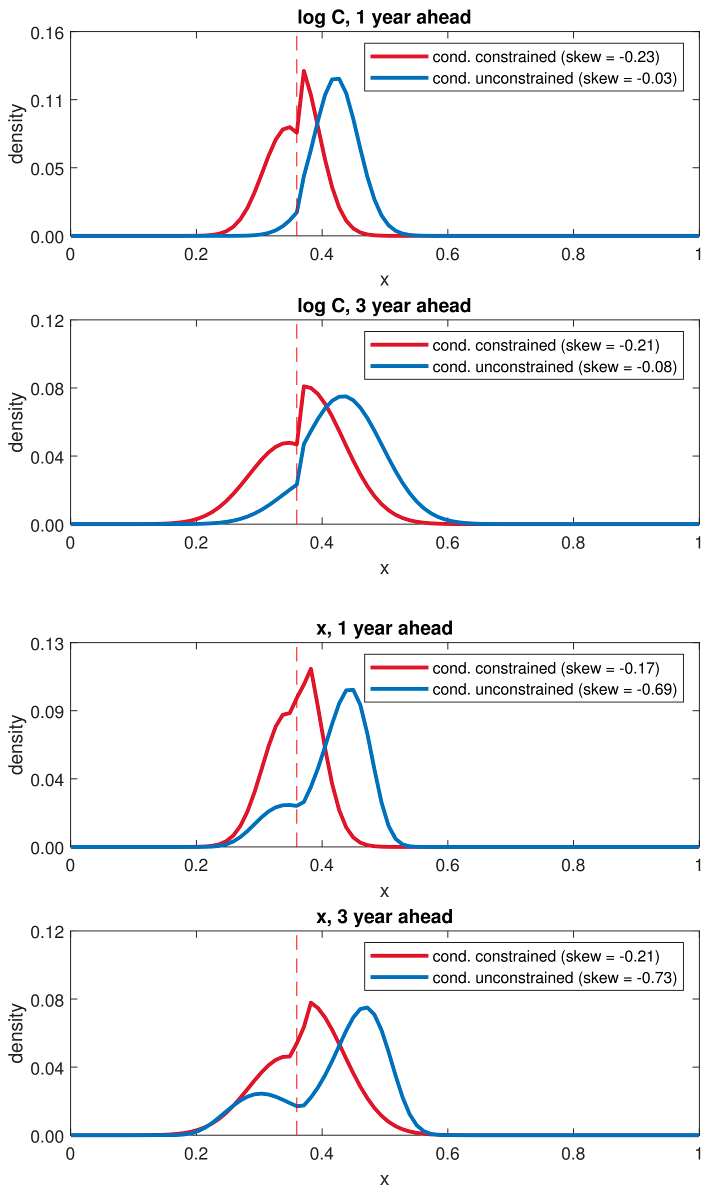
NOTE: The upper-left panel shows the term premium of zero-coupon bonds of different maturities. The upper-right panel shows the term structure of the term premium when the state variable x is at its unconditional mean. The bottom-left panel shows the estimated coefficients of the Fama-Bliss regressions (shown in the main text) for simulated data from the nominal model. The simulation consists of 200 paths with 100,000 realizations in each path. The estimated coefficients from the data come from running the Fama-Bliss regression (17) using nominal yields in the period 1971-2018 (see the appendix for details about the data). The confidence intervals are two standard deviations from the point estimate. The lower-right panel shows the 5-year term premium and the forward-spot spread in the model.

FIGURE 6. The Role of Risk Aversion and the EIS



NOTE: The left panel shows the slope of the real yield curve (10-year minus 1-year spread) for different parametrizations of the risk aversion and EIS coefficients. The right panel shows the real rate across the state space for different parametrizations of the risk-aversion and EIS coefficients, rescaled such that the real rate is the same in the first best for all parametrizations. The baseline calibration is the one reported in Table 1.

FIGURE 7. Term Structure of Conditional Distributions for Consumption



NOTE: This figure shows (1) the conditional distributions of consumption 1 and 3 years ahead (top two panels) and (2) the conditional distributions of the endogenous state variable x 1 and 3 years ahead (bottom two panels). The distributions are conditional on the state of the economy being an unconstrained one (blue lines) or a constrained one (red lines). The red dashed line is the point in the state space at which constraints bind.

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7 Appendix

Proof of Proposition 1. I derive the law of motion of the endogenous state variable, x_t, which is key in solving the ODEs. Applying Itô's lemma to $\frac{n_{f,t}}{q_t}$:

$$\mathrm{d}x_t = \frac{\mathrm{d}n_{f,t}}{q_t} - \frac{n_{f,t}}{q_t^2}\mathrm{d}q_t + \frac{n_{f,t}}{q_t^3}\left(\mathrm{d}q_t\right)^2 - \frac{\mathrm{d}q_t}{q_t^2}\mathrm{d}n_{f,t} + \frac{\lambda}{q_t}(\overline{x}q_t - n_{f,t})\mathrm{d}t, \tag{18}$$

where the last term, $\frac{\lambda}{q_t}(\overline{x}q_t - n_{ft})$, represents the initial capital received by financiers, net of their aggregate dividends (and divided by q_t). Notice that, on the one hand, financiers receive a fraction $\overline{x}$ of total wealth q_t each instant, with intensity λ, as a start-up capital to initiate their business. Hence, financiers' aggregate wealth increases $\lambda\overline{x}q_t$ each instant. On the other hand, financiers pay their wealth as dividends, $n_{f,t}$, with intensity λ, which decreases financiers' aggregate wealth by $\lambda n_{f,t}$ each instant. Combining these two terms, the net change in aggregate wealth is given by $\lambda(\overline{x}q_t - n_{ft})$, which I divide by q_t as a result of Itô's lemma taken on x_t. Finally, notice that $\frac{\lambda}{q_t}(\overline{x}q_t - n_{ft}) = \lambda(\overline{x} - x_t)$.

Then, substituting the law of motion for q_t and $n_{f,t}$ in (18), I get

$$\begin{aligned}\mathrm{d}x_t &= x_t\left[r_t + \frac{q_t\theta_{f,t}}{n_{f,t}}\left(\mathbb{E}_t\left[\mathrm{d}R_{f,t}\right] - r_t\right) - \mu_{q,t} + \left(\frac{q_t\theta_{f,t}}{n_{f,t}} - 1\right)\sigma_{q,t}^2\right]\mathrm{d}t + \\ &\quad x_t\left(\frac{q_t\theta_{f,t}}{n_{f,t}} - 1\right)\sigma_{q,t}\mathrm{d}W_t + \lambda\left(\overline{x} - x_t\right)\mathrm{d}t.\end{aligned}$$

Based on the previous expression, I denote the drift and diffusion as

$$\begin{aligned}x_t\mu_{x,t} &= x_t\left[r_t + \frac{q_t\theta_{f,t}}{n_{f,t}}\left(\mathbb{E}_t\left[\mathrm{d}R_{f,t}\right] - r_t\right) - \mu_{q,t} + \left(\frac{q_t\theta_{f,t}}{n_{f,t}} - 1\right)\sigma_{q,t}^2\right] + \lambda\left(\overline{x} - x_t\right), \\ x_t\sigma_{x,t} &= x_t\left(\frac{q_t\theta_{f,t}}{n_{f,t}} - 1\right)\sigma_{q,t}.\end{aligned}$$

I next derive the system of ordinary differential equations for five unknown functions—namely, the price-dividend ratio $p(x)$, the financiers' marginal value of wealth $\phi(x)$, the savers' utility function $\xi(x)$, the real bond prices $\{P(x,\tau)\}_{\tau\geq 0}$, and the nominal bond prices $\left\{P^{\$}(x,s,\tau)\right\}_{\tau\geq 0}$.

When the financing constraint is not binding the first-order conditions of financiers' problem is

$$\mathbb{E}_t\left[\mathrm{d}R_{f,t}\right] - r_t\mathrm{d}t + \mathbb{E}_t\left[\left(\frac{\mathrm{d}m_t}{m_t} + \frac{\mathrm{d}\phi_t}{\phi_t}\right)\mathrm{d}R_{f,t}\right] = 0. \tag{19}$$

Equation (19) is the first equation in proposition 2 (i.e., the ODE for the price-dividend in the unconstrained region). Then, substituting (19) into financiers' value function (9), yields the second equation in the system (i.e., the ODE for financier's marginal value of wealth, ϕ). When financiers are constrained, the savers' pricing equation is

$$\mathbb{E}_t\left[\mathrm{d}R_{s,t}\right] - r_t\mathrm{d}t + \mathbb{E}_t\left[\frac{\mathrm{d}m_t}{m_t}\mathrm{d}R_{s,t}\right] = 0. \tag{20}$$

Equation (20) is the fourth equation in proposition 2, and replacing (20) into financiers' value function gives the fifth equation.

Finally, the third and sixth equations are the savers' value function. I denote the value function with optimal policies as (c^*, U^*)

$$0 = \frac{1}{1-\frac{1}{\psi}}\left\{\rho\left(c^*\right)^{1-\frac{1}{\psi}}\left[(1-\gamma)U^*\right]^{\frac{\frac{1}{\psi}-\gamma}{1-\gamma}} - \rho(1-\gamma)U^*\right\} + E_t\left[\mathrm{d}U^*\right].$$

I guess and verify the solution is given by

$$U^* = \frac{\left(\xi(x)\,c^*\right)^{1-\gamma}}{1-\gamma}. \tag{21}$$

Then, using Itô's lemma and substituting, I get

$$0=\frac{\rho}{1-\frac{1}{\psi}}\left\{\xi^{\frac{1}{\psi}-1}-1\right\}+\mu_c-\frac{\gamma}{2}\sigma_c^2+\frac{\xi_x}{\xi}x\mu_x-\frac{\gamma}{2}\left(\frac{\xi_{xx}}{\xi}x\sigma_x\right)^2+(1-\gamma)\frac{\xi_x}{\xi}x\sigma_c. \qquad (22)$$

All drifts and diffusions terms in proposition 2, terms $\mu_p(x),\mu_\phi(x),\mu_\xi(x),\mu_P(x,\tau),\mu_c(x)$ and $\sigma_p(x),\sigma_\phi(x),\sigma_\xi(x),\sigma_P(x,\tau),\sigma_c(x)$ are partial derivatives from applying Itô's lemma in their corresponding functions. That is,

$$\begin{aligned}
\mu_p(x) &= \frac{p_x}{p}\mathbb{E}_t[\mathrm{d}x]+\frac{1}{2}\frac{p_{xx}}{p}\mathbb{E}_t\left[\mathrm{d}x^2\right],\\
\mu_\phi(x) &= \frac{\phi_x}{\phi}\mathbb{E}_t[\mathrm{d}x]+\frac{1}{2}\frac{\phi_{xx}}{\phi}\mathbb{E}_t\left[\mathrm{d}x^2\right],\\
\mu_\xi(x) &= \frac{\xi_x}{\xi}\mathbb{E}_t[\mathrm{d}x]+\frac{1}{2}\frac{\xi_{xx}}{\xi}\mathbb{E}_t\left[\mathrm{d}x^2\right],\\
\mu_P(x,\tau) &= \frac{P(\tau)_x}{P(\tau)}\mathbb{E}_t[\mathrm{d}x]+\frac{1}{2}\frac{P(\tau)_{xx}}{P(\tau)}\mathbb{E}_t\left[\mathrm{d}x^2\right],
\end{aligned}$$

and

$$\sigma_p(x)=\frac{p_x}{p}x\sigma_x;\sigma_\xi(x)=\frac{\xi_x}{\xi}x\sigma_x;\sigma_\xi(x)=\frac{\xi_x}{\xi}x\sigma_x;\sigma_P(x,\tau)=\frac{P(\tau)_x}{P(\tau)}x\sigma_x.$$

The drift and diffusion for the nominal bond prices can be expressed in a similar way, in terms of the partial derivatives, but include the extra terms associated with of the exogenous state variable—the monetary policy shocks.

For consumption, use the market clearing condition for goods

$$\begin{aligned}
c_t &= \left[\omega\alpha_t^s(1-x_t)+x_t\alpha_t^f\right]Y_t,\\
&= \left[\omega+(1-\omega)x_t\alpha_t^f\right]Y_t,
\end{aligned}$$

and apply Itô's lemma:

$$\mu_c = \frac{(1-\omega)\, x_t \alpha_t^f}{\left[\omega + (1-\omega)\, x_t \alpha_t^f\right]} \left[\mu_x + \frac{\phi_x}{\phi} x \mu_x + \frac{1}{2}\frac{\phi_{xx}}{\phi}(x\sigma_x)^2 + \frac{\phi_x}{\phi} x \sigma_x^2 + \left(\frac{\phi_x}{\phi} x + 1\right)\sigma_x \sigma\right] + \mu,$$
$$\sigma_c = \frac{(1-\omega)\, x_t \alpha_t^f}{\left[\omega + (1-\omega)\, x_t \alpha_t^f\right]} \left(\frac{\psi_x}{\psi} x + 1\right)\sigma_x + \sigma.$$

Finally, I show $r_t \mathrm{d}t = -\mathbb{E}_t\left[\frac{\mathrm{d}m_t}{m_t}\right]$. For this, I follow the characterization in Cox, Ingersoll and Ross (1985a) and Duffie and Epstein (1992a). The savers' problem is

$$0 = \max_{c,\theta_s} \frac{1}{1-\frac{1}{\psi}} \left\{\rho c^{1-\frac{1}{\psi}} \left[(1-\gamma)\, U\right]^{\frac{\frac{1}{\psi}-\gamma}{1-\gamma}} - \rho\,(1-\gamma)\, U\right\} + \mathbb{E}_t\left[\mathrm{d}U\right]. \qquad (23)$$

subject to (2). I apply Itô's lemma on $U(x,n)$ to write

$$\mathbb{E}_t\left[\mathrm{d}U\right] = U_x \mathbb{E}_t\left[\mathrm{d}x\right] + \frac{1}{2} U_{xx} \mathbb{E}_t\left[\mathrm{d}x^2\right] + U_n \mathbb{E}_t\left[\mathrm{d}n\right] + \frac{1}{2} U_{nn} \mathbb{E}_t\left[\mathrm{d}n^2\right] + U_{nx} \mathbb{E}_t\left[\mathrm{d}n\mathrm{d}x\right].$$

The first-order conditions are

$$\rho c^{-\frac{1}{\psi}} \left[(1-\gamma)\, U\right]^{\frac{\frac{1}{\psi}-\gamma}{1-\gamma}} = U_n,$$
$$q U_n \left(\mathbb{E}_t\left[\mathrm{d}R_s\right] - r\right) + U_{nn}\theta_s q^2 \sigma_q^2 + U_{xn} q \sigma_q \sigma_x = 0.$$

Then, I substitute the law of motion for n, and the first-order conditions in (23) to get

$$0 = \frac{\rho^\psi}{1-\frac{1}{\psi}} \left[(1-\gamma)\, U\right]^{\frac{\frac{1}{\psi}-\gamma}{1-\gamma}\psi} U_n^{1-\psi} - \frac{\rho\,(1-\gamma)\, U}{1-\frac{1}{\psi}} - \rho^\psi \left[(1-\gamma)\, U\right]^{\frac{\frac{1}{\psi}-\gamma}{1-\gamma}\psi} U_n^{1-\psi}$$
$$+ U_n\left[T + rn\right] - \frac{1}{2} U_{nn} \left(\theta^s q\right)^2 \sigma_q^2 + U_x \mathbb{E}_t\left[\mathrm{d}x\right] + \frac{1}{2} U_{xx} \mathbb{E}_t\left[\mathrm{d}x^2\right]. \qquad (24)$$

The next step is to take the derivative of (24) with respect to n. After some algebra,

$$\begin{aligned} 0 \; = \; & \left[\left(\frac{\frac{1}{\psi} - \gamma}{1-\gamma} \right) \left(\frac{\psi}{\psi - 1} \right) \frac{U_n}{U} - \frac{U_{nn}}{U_n} \right] \rho^{\psi} \left[(1-\gamma) U \right]^{\frac{\frac{1}{\psi}-\gamma}{1-\gamma}\psi} U_n^{1-\psi} \\ & - \frac{\rho (1-\gamma)}{1 - \frac{1}{\psi}} U_n + U_{nn} \left[T + rn \right] + U_n r \\ & - \frac{1}{2} U_{nnn} \left(\theta^s q \right)^2 \sigma_q^2 + U_{nx} \mathbb{E}_t \left[\mathrm{d}x \right] + \frac{1}{2} U_{nxx} \mathbb{E}_t \left[\mathrm{d}x^2 \right]. \end{aligned} \tag{25}$$

The final step is to subtract the stochastic discount factor (SDF) from the expression above. For this, I follow Duffie and Epstein (1992a), and express

$$\frac{\mathrm{d}m_t}{m_t} = \frac{\mathrm{d}f_c}{f_c} + f_U \, \mathrm{d}t,$$

with

$$f_U \; = \; \frac{\left(\frac{1}{\psi} - \gamma \right) \rho}{1 - \frac{1}{\psi}} c^{1-1/\psi} \left[(1-\gamma) U \right]^{\frac{\frac{1}{\psi}-\gamma}{1-\gamma} - 1} - \frac{\rho (1-\gamma)}{1 - \frac{1}{\psi}}, \tag{26}$$

$$f_c \; = \; \rho c^{-\frac{1}{\psi}} \left[(1-\gamma) U \right]^{\frac{\frac{1}{\psi}-\gamma}{1-\gamma}}. \tag{27}$$

Using the first-order condition, $f_c = U_n$, the conditional expectation of the SDF can be written as

$$\begin{aligned} \mathbb{E}_t \left[\frac{\mathrm{d}m}{m} \right] \; = \; & \frac{U_{nn}}{U_n} \left[rn + T \right] - \rho^{\psi} \left[(1-\gamma) U \right]^{\frac{\frac{1}{\psi}-\gamma}{1-\gamma}\psi} U_n^{-\psi} \frac{U_{nn}}{U_n} - \frac{1}{2} \frac{U_{nnn}}{U_n} \left(\theta q \right)^2 \sigma_q^2 \\ & + \frac{U_{nx}}{U_n} \mathbb{E}_t \left[\mathrm{d}x \right] + \frac{1}{2} \frac{U_{nxx}}{U_n} \mathbb{E}_t \left[\mathrm{d}x^2 \right] \\ & + \frac{\frac{1}{\psi} - \gamma}{1 - \frac{1}{\psi}} \rho^{\psi} \left[(1-\gamma) U \right]^{\frac{(1-\psi)\gamma}{1-\gamma}} U_n^{1-\psi} - \frac{\rho (1-\gamma)}{1 - \frac{1}{\psi}}. \end{aligned} \tag{28}$$

Finally, I divide (25) by U_n and subtract the resulting expresion from (28) to get $\mathbb{E}_t\left[\frac{\mathrm{d}m}{m}\right] = -r_t \mathrm{d}t$.

$\overline{x}$ **as a Tax Rate**. The role of $\overline{x}$ is to avoid a degenerate invariant wealth distribution in which financiers accumulate all the wealth. I next discuss an intuition to interpret $\overline{x}$ as a tax rate. As noted in the text, financiers pay to savers $\lambda n_{f,t}$ each instant. Suppose a government taxes dividends and redistribute them to financiers (motivated by the fact that financiers can achieve a better allocation of goods and capital since they are more efficient than savers). Let the tax rate be $\overline{x}$, so that the net dividend received by savers is $(1-\overline{x})\lambda n_{f,t}$ and the government—who runs a balanced budget—rebates the tax to financiers, who receive $\overline{x}\lambda n_{f,t}$. On the other hand, assume new financiers must have $\overline{x} n_{s,t}$ resources to start a financial firm, which can be motivated by a regulation. Thus, financiers' total wealth will receive $\lambda \overline{x} n_{s,t}$ from incoming financiers. Then, aggregate wealth from financiers would have the following terms

$$\mathrm{d}n_{f,t} = -\lambda n_{f,t}\mathrm{d}t + \left(\lambda \overline{x} n_{f,t} + \lambda \overline{x} n_{s,t}\right)\mathrm{d}t + \text{others},$$

where $-\lambda n_{f,t}$ are the aggregate dividends paid by financiers to savers; $\lambda \overline{x} n_{f,t}$ are the taxes on dividends paid by savers and rebated to financiers; and $\lambda \overline{x} n_{s,t}$ is the minimum capital needed to start the financial firm. Then, when computing Itô's lemma on $x_t = n_{f,t}/q_t$, we have

$$\begin{aligned} \mathrm{d}x_t &= -\lambda \frac{n_{f,t}}{q_t}\mathrm{d}t + \frac{1}{q_t}\left(\lambda \overline{x} n_{f,t} + \lambda \overline{x} n_{s,t}\right)\mathrm{d}t + \text{others}, \\ &= -\lambda \frac{n_{f,t}}{q_t}\mathrm{d}t + \frac{\lambda \overline{x}}{q_t}\left(n_{f,t} + n_{s,t}\right)\mathrm{d}t + \text{others}, \\ &= \lambda\left(\overline{x} - x_t\right) + \text{others}, \end{aligned} \tag{29}$$

where the last step uses $n_{f,t} + n_{s,t} = q_t$ and $\frac{n_{f,t}}{q_t} = x_t$. Note that expression (29) is the same as expression (18).

The Real Interest Rate in the Frictionless Case. When there are no financial frictions, the interest rate is constant. The consumption process follows a geometric Brownian motion. Using expressions (26) and (27), as well as the value function, together with the definition $-\mathbb{E}_t\left[\frac{\mathrm{d}m}{m}\right] = r_t \mathrm{d}t$, the interest rate in the frictionless case is

$$r = \rho + \frac{1}{\psi}\mu - \frac{1}{2}\left(1 + \frac{1}{\psi}\right)\gamma\sigma^2.$$

Boundary Conditions. The boundary conditions for the system of three second-order ordinary differential equations are

$$\lim_{x \nearrow 1} \phi_x = \lim_{x \nearrow 1} \phi_{xx} = \lim_{x \nearrow 1} q_x = \lim_{x \nearrow 1} q_{xx} = \lim_{x \nearrow 1} \xi_x = \lim_{x \nearrow 1} \xi_{xx} = 0.$$

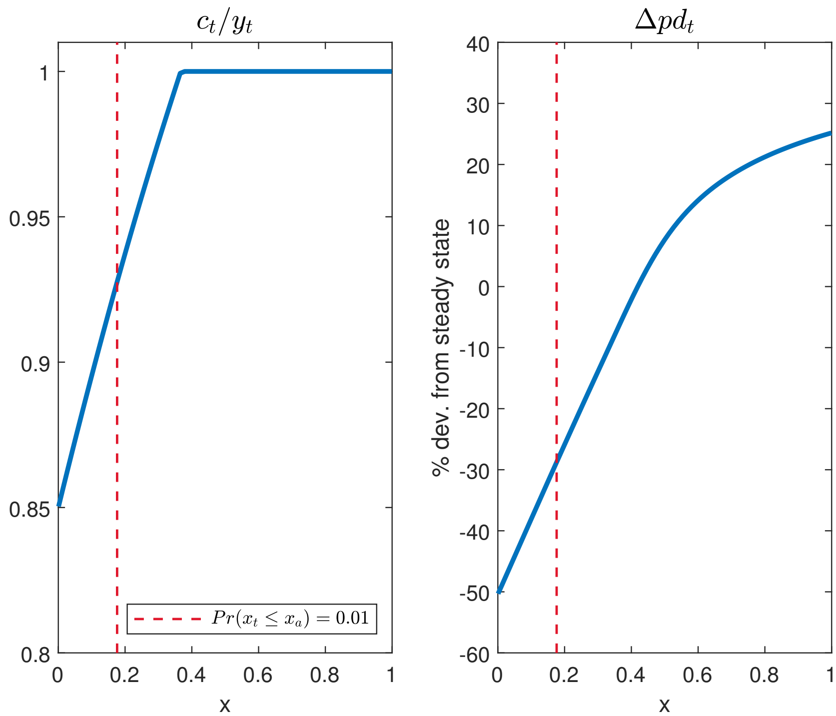
Intuitively, these boundary conditions imply that, as x approaches 1 and financiers are well capitalized, the economy converges smoothly to the first best and there is no amplification of aggregate risk (i.e., asset price volatility is equal to fundamental volatility, so $\sigma_q = \sigma$). Note, however, that x never reaches 1 because, as x increases towards 1, the drift of x becomes negative (due to the dividend payments) and the diffusion component approaches zero.[24]

Consumption and Price-Dividend Ratio During Financial Disruptions. Figure (**??**) below shows the consumption losses (left panel) and changes in the price-dividend ratio (right panel) across the state space. The vertical red dashed line shows the point in the

[24] Karlin and Taylor (1981) provide further details about the boundary behavior of stochastic processes with reflecting boundaries.

state space at which the invariant distribution accumulates is 1% probability.

FIGURE A.1. Consumption and Price-dividend ratio



As shown in figure A.1, it is very unlikely that, in the model's equilibrium, consumption losses will exceed -7% and the price-dividend ratio changes exceed -28.8% in a given year. Muir (2017) shows that declines in consumption and the price-dividend ratio associated with financial crises—defined as a banking panic or banking crisis—are broadly in line with the model: The evidence for financial crises indicates that those episodes are associated with an average decline of approximately 25% in price-dividend ratio and a 9% decline in consumption levels.[25] Additionally, Greenwood et al. (2022) show, using a very similar dataset than Muir (2017), that the unconditional probability of a financial crises is 4%.[26] Hence, the evidence points to rare occasions (4% prob-

[25] More details about the definition of financial crises can be found in Muir (2017), Section 2.
[26] Jordà, Schularick and Taylor (2016)

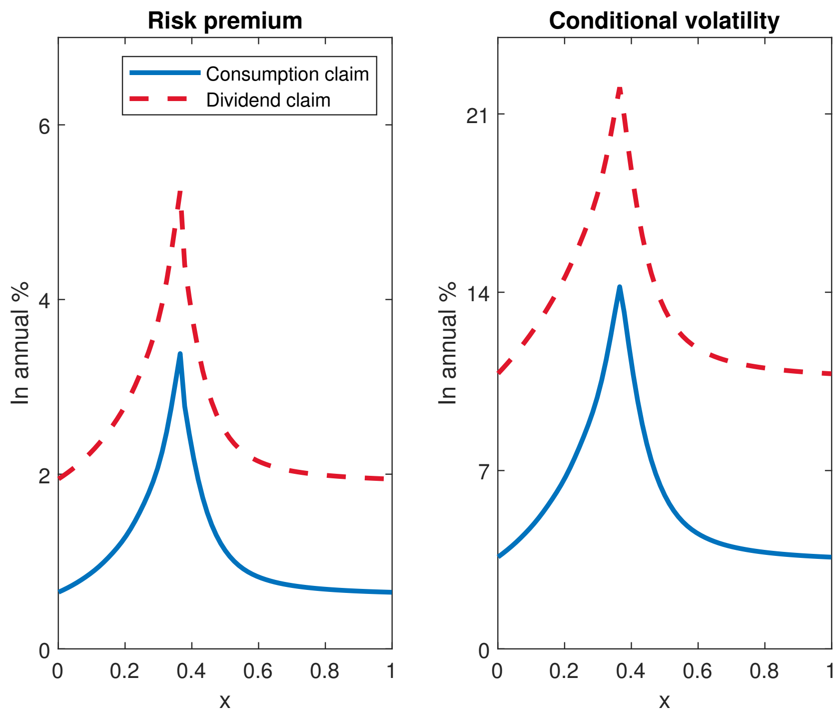
ability) in which financial intermediaries wealth is unpaired, and those occasions are characterized by approximately 9% consumption losses and a 25% decline in the price-dividend ratio, which is similar to the model implications.

Consumption and Dividend Claims: Risk premium and Volatility. Figure A.2 shows the risk premium and the (conditional) volatility of the consumption claim and a dividend claim. I compute the dividend claim as a leveraged claim on consumption, as it is common in the asset pricing literature (Abel (1990),Campbell (2003), Campbell et al. (2020), among others). That is, the dividend claim is computed using the dividend process $D_t = Y_t^{\eta}$, which, using Itô's lemma, follows

$$\begin{aligned} \frac{\mathrm{d}D_t}{D_t} &= \eta \frac{\mathrm{d}Y_t}{Y_t} + \frac{1}{2}\eta\left(\eta - 1\right)\left(\frac{\mathrm{d}Y_t}{Y_t}\right)^2, \\ &= \left[\eta\mu + \frac{1}{2}\eta\left(\eta-1\right)\sigma^2\right]\mathrm{d}t + \eta\sigma\mathrm{d}W_t. \end{aligned}$$

I use $\eta = 3$, a value that is consistent with the asset pricing literature (e.g., Bansal and Yaron (2004)).

FIGURE A.2. Consumption and dividend claims: Risk premium and volatility



NOTE: The left panel shows the risk premium on the consumption claim (blue solid line) and on the dividend claim (red dashed line). The right panel shows theconditional volatility for the consumption claim (blue solid line) and for the dividend claim (red dashed line). The dividend claim follows $D_t = Y_t^\eta$, using $\eta = 3$.

TABLE A.2. Summary Statistics on TIPS and Nominal Treasuries.

		1y	2y	3y	5y	7y	10y
TIPS							
	Average	0.016	0.017	0.018	0.020	0.021	0.023
	St. Dev.	0.019	0.017	0.016	0.015	0.013	0.012
Nominal							
	Average	0.052	0.054	0.056	0.059	0.062	0.064
	St. Dev.	0.035	0.034	0.033	0.032	0.030	0.029

NOTE: The source of data for real yields is Chernov and Mueller (2012) from 1971-2002 and Gürkaynak et al. (2010) for TIPS from 2003-2018. The data from Chernov and Mueller (2012) can be easily accessed on the Muller's website. The data for nominal yields in Figure 4 are from Gürkaynak et al. (2007) from 1971-2018. The table shows the mean and standard deviation of real and nominal yields, which are the statistics used in figures 2 and 4. The summary statistics are expressed in decimals at an annual frequency.

Numerical Method. I use a spectral collocation method based on Chebyshev polynomials of the first kind to numerically solve the model. Conceptually, the numerical solution consists of representing the unknown functions as Chebyshev polynomials on a grid and then substituting them into the ODEs that characterize the equilibrium. In particular, I solve for: i) The financiers' value functions; ii) the price-dividend ratio; iii) the savers' value function; iv) real bond prices; and v) nominal bond prices. The system of equations is in proposition 1 in the main text and the detailed derivation is in the appendix.

The steps are as follow:

1. First, construct a grid with K Chebyshev nodes

   $$h_i = \cos\left(\frac{2i+1}{2\left(K+1\right)}\pi\right),\ i = 0, ..., K.$$

   Therefore, $h_i \in \left[-1, 1\right]$. Since, in the model $x \in (0, 1)$, I express the grid on x as $x_i = \frac{1}{2}\left(1 + h_i\right)$.

2. Then, a given function $g(x)$, which needs to be solved, can be written in polynomial form

   $$g(x) = \sum_{i=0}^{K} a_i \Psi_i \left(h_i\left(x\right)\right) + O\left(K\right),$$

   where K is the order of the polynomial, Ψ is the basis function (which, in this case, is the Chebyshev polynomials), $\{a_i\}_{i=0}^{K}$ are unknown coefficients that need to solved, h_i are the Chebyshev nodes, and $O\left(K\right)$ is an approximation error.

3. Then, solve for the associated set of unknown coefficients $\{a_i\}_{i=0}^{K}$ in each function, such that equilibrium conditions are verified.

(a) Start with a guess for the unknown coefficients.

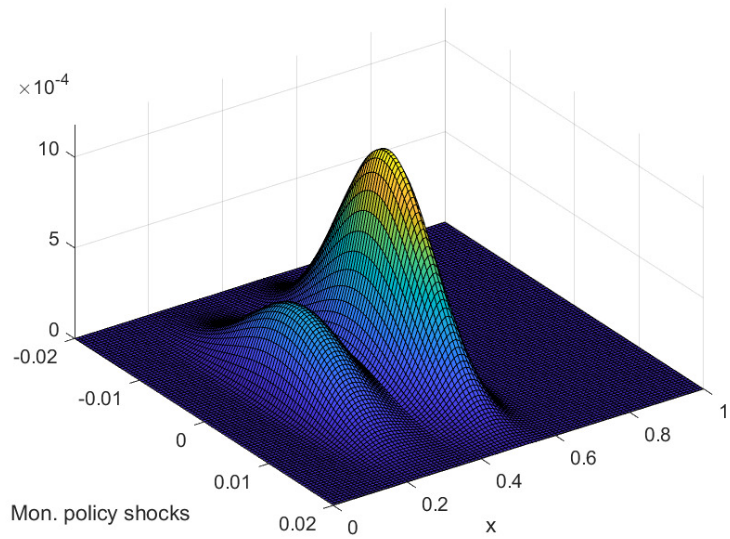
(b) For a guessed solution, use financiers' optimality condition to compute x^*, which is the point at which the leverage constraint binds. That is, the point at which $\alpha_{f,t} = 1/x_t$ becomes $\alpha_{f,t} = \phi_t/\kappa$.

(c) Solve the corresponding system of equations with a nonlinear solver and verify.

4. Once the equilibrium is solved, I solve the yield curve using forward finite difference across the maturity dimension. That is, starting from $P^{(0)}(x) = 1$, I solve for the bond prices in the state space (using the steps above) and iterate across the maturity dimension with

$$\frac{P^{(\tau+\Delta)}(x) - P^{(\tau)}(x)}{\Delta} \approx P_\tau^{(\tau)}(x).$$

FIGURE A.3. Bi-variate distribution of x and monetary policy shocks.



NOTES: This figure shows the invariant distribution of the endogenous state variable, x, and the exogenous state variable, monetary policy shocks.